Chapter Five
Energy of a System

5-1- Work Done by a Constant Force

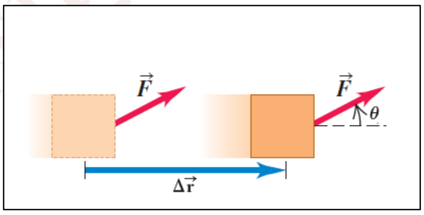
The work done on a system by a constant force $\vec{F}$ is defined to be the product of the magnitude of the force, the magnitude of the displacement of the point of application of the force, and the cosine of the angle between the force $\vec{F}$ and the displacement vectors $\Delta\vec{r}$.

$$W = F\Delta r\cos\theta = \vec{F}\cdot\Delta\vec{r} \qquad \text{.........(1)}$$

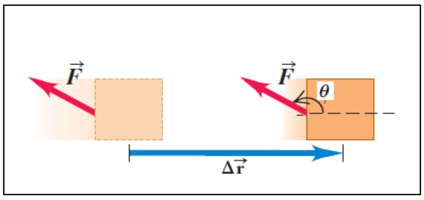
Work is a scalar quantity and its SI unit is the N·m. One N·m = 1 joule (J).

Work is energy transferred to or from an object by means of a force acting on the object.

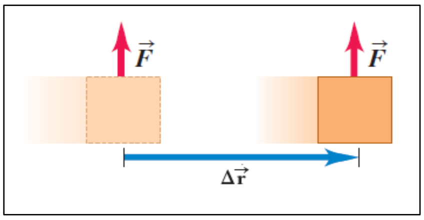
If W is positive, energy is transferred to the system and we say the work is done on the system.



If W is negative, energy is transferred from the system and we say the work is done by the system.



There is no work (W=0) if the force is perpendicular to the direction of the displacement.



Ex: A man cleaning a floor pulls a vacuum cleaner with a force of magnitude 50 N at an angle of 30° with the horizontal. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3 m to the right.

Soln.

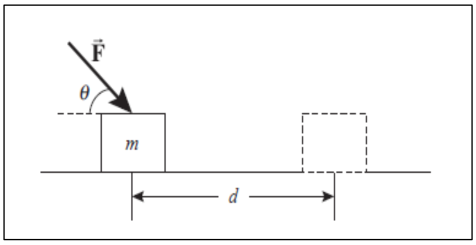
$$W = F\Delta r\cos\theta = (50)(3)\cos(30^o) = 130\,J$$

Ex: A particle moving in the xy-plane undergoes a displacement given by $\Delta\vec{r} = (2\hat{i} + 3\hat{j})$ m as a constant force $\vec{F} = (5\hat{i} + 2\hat{j})$ N acts on the particle. Calculate the work done by $\vec{F}$ on the particle.

Soln:

$$W = \vec{F}\cdot\Delta\vec{r} = (5\hat{i} + 2\hat{j})\cdot(2\hat{i} + 3\hat{j}) = (5\hat{i}\cdot 2\hat{i}) + (5\hat{i}\cdot 3\hat{j}) + (2\hat{j}\cdot 2\hat{i}) + (2\hat{j}\cdot 3\hat{j}) = 10 + 0 + 0 + 6 = 16\,J$$

Ex: A block of mass 2.5 kg is pushed a distance 2.2 m along a frictionless horizontal table by a constant applied force of magnitude 16 N directed at an angle 25° above the horizontal as shown in figure below. Determine the work done on the block by (a) the applied force, (b) the normal force exerted by the table, (c) the gravitational force, and (d) the net force on the block.



Soln:

a) By the applied force → $W_{app} = Fd\cos\theta = (16\text{ N})(2.2\text{ m})\cos(25^o) = 31.9\text{ J}$

b) By the normal force → $W_n = nd\cos\theta = (F\sin\theta + mg)(d)\cos(90^o) = 0$

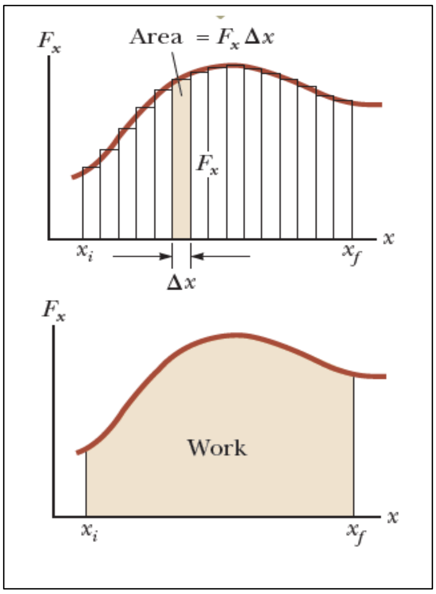
c) By the gravitational force → $W_g = F_g d\cos\theta = (mg)(d)\cos(90^o) = 0$

d) Net work done on the block → $W_{net} = W_{app} + W_n + W_g = 31.9\text{ J}$

5-2- Work done by a varying force

Consider a particle being displaced along the x-axis under the action of a force that varies with position. The particle is displaced in the direction of increasing x from $x=x_i$ to $x=x_f$. In such a situation, we cannot use $W = F\Delta r\cos\theta = \vec{F}\cdot\Delta\vec{r}$ to calculate the work done by the force because this relationship applies only when $\vec{F}$ is constant in magnitude and direction.

To find the work done by this force, we divide the total displacement into small segments of width Δx, the work done by the force during segment Δx is approximately



$$W \approx F_x \Delta x$$

The work done by the force in the total displacement from $x=x_i$ to $x=x_f$ is

$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$

In the limit that the number of segments becomes very large and the width of each becomes very small

$$\lim_{\Delta x\to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

$$W = \int_{x_i}^{x_f} F_x dx \qquad \ldots\ldots\ldots(2)$$

If more than one force acts on a system, the total work (net work) done on the system is just the work done by the net force

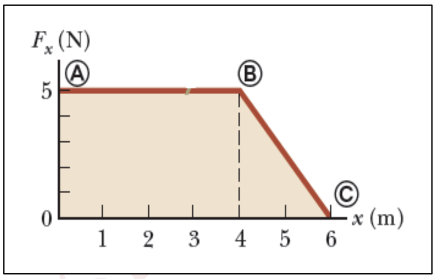
$$W_{net} = \int_{x_i}^{x_f} \left(\sum F_x\right) dx$$

Ex: A force acting on a particle varies with *x* as shown in figure below. Calculate the work done by the force on the particle as it moves from $x=0$ to $x=6$ m.
Soln:

$$W_{Ⓐ\text{ to }Ⓑ} = (5.0\text{ N})(4.0\text{ m}) = 20\text{ J}$$

$$W_{Ⓑ\text{ to }Ⓒ} = \tfrac{1}{2}(5.0\text{ N})(2.0\text{ m}) = 5.0\text{ J}$$

$$W_{Ⓐ\text{ to }Ⓒ} = W_{Ⓐ\text{ to }Ⓑ} + W_{Ⓑ\text{ to }Ⓒ} = 20\text{ J} + 5.0\text{ J} = 25\text{ J}$$



5-3- Work done by a spring

A model of a common physical system for which the force varies with position is shown in figure below. If the spring is either stretched or compressed a small distance from its unstretched (equilibrium) configuration, it exerts on the block a force that can be expressed as

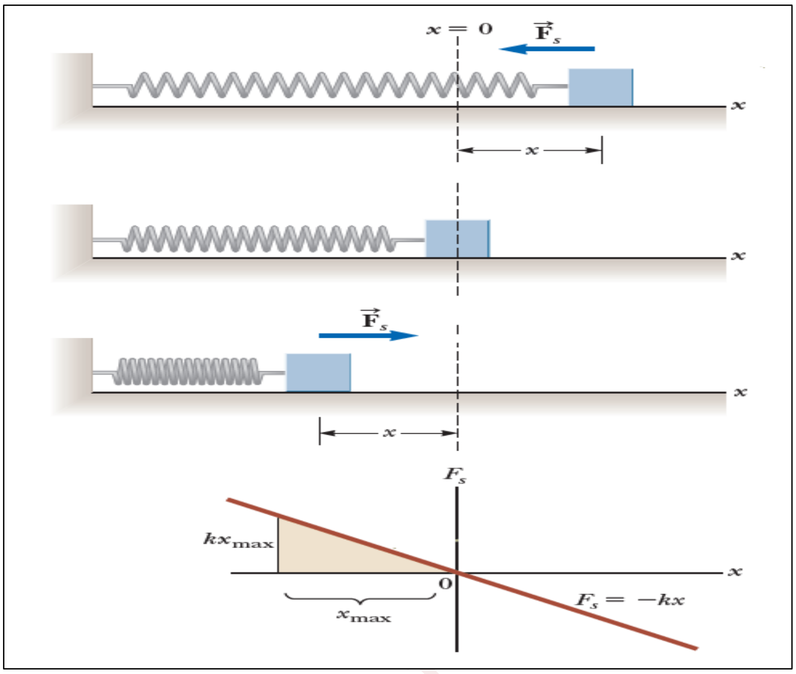
$$F_s = -kx \qquad \text{(Hooke's law)} \qquad \ldots\ldots\ldots(3)$$

where x is the position of the block relative to its equilibrium (x=0) position and k is a positive constant called the force constant or the spring constant of the spring. This force law for springs is known as **Hooke's law**. The value of k is a measure of the stiffness of the spring. Stiff springs have large k values, and soft springs have small k values. As can be seen from equation (3), the units of k are N/m. Because the spring force always acts toward the equilibrium position (x=0), it is sometimes called a restoring force.
The work W_s done on the object by the spring force when the object is moved from an initial position x_i to a final position x_f is:

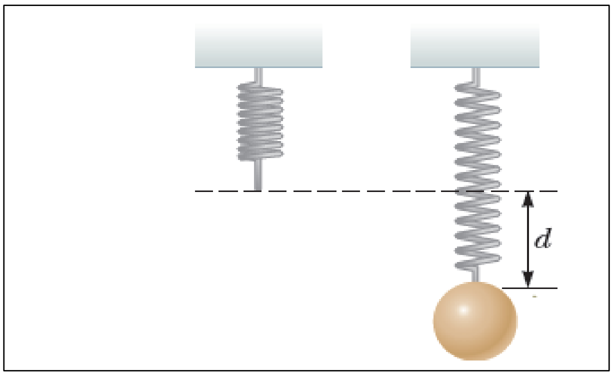
$$W_s = \int_{x_i}^{x_f} F_s dx = \int_{x_i}^{x_f} (-kx) dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 \qquad \ldots\ldots\ldots(4)$$

The work W_s done by the spring force on the block as the block moves from $x_i = -x_{max}$ to $x_f = 0$ is:

$$W_s = \int_{x_i}^{x_f} F_s dx = \int_{-x_{max}}^{0} (-kx) dx = \frac{1}{2}kx_{max}^2 \qquad \ldots\ldots\ldots(5)$$



Ex: A spring is hung vertically, and an object of mass *m* is attached to its lower end. Under the action of the load *mg*, the spring stretches a distance *d* from its equilibrium position.
(A) If a spring is stretched 2 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?
(B) How much work is done by the spring on the object as it stretches through this distance?
(C) Evaluate the work done by the gravitational force on the object.



Soln:

(A) $\sum F_y = 0 \rightarrow F_s - mg = 0$

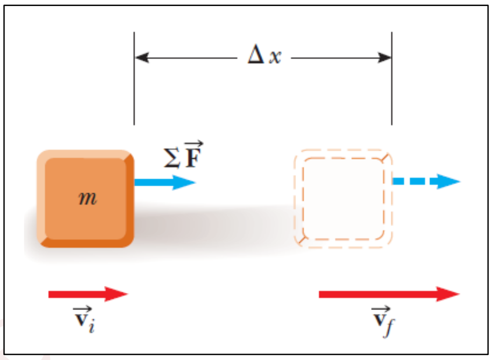
$$k = \frac{mg}{x} = \frac{mg}{d} = \frac{0.55 \times 9.8}{2 \times 10^{-2}} = 2.7 \times 10^2 \text{ N/m}$$

(B) $$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = 0 - \frac{1}{2}kd^2$$
$$= -\frac{1}{2}(2.7 \times 10^2)(2 \times 10^{-2}) = -5.4 \times 10^{-2} \text{ J}$$

(C) $$W = \vec{F} \cdot \Delta\vec{r} = mgd\cos(0) = (0.55)(9.8)(2 \times 10^{-2}) = 1.1 \times 10^{-1} \text{ J}$$

5-4- Kinetic energy and the work–kinetic energy theorem

Consider a system consisting of a single object. The figure below shows a block of mass m moving through a displacement directed to the right under the action of a net force $\sum \vec{F}$, also directed to the right. We know from Newton's second law that the block moves with an acceleration $\vec{a}$. If the block moves through a displacement $\Delta\vec{r} = \Delta x\hat{i} = (x_f - x_i)\hat{i}$, the net work done on the block by the net force $\sum \vec{F}$ is:



$$W = \int_{x_i}^{x_f} \left(\sum F_x\right) dx$$

Using Newton's second law, we can substitute for the magnitude of the net force $\sum \vec{F} = m\vec{a}$, and then perform the following chain-rule manipulations on the integrand:

$$W = \int_{x_i}^{x_f} madx = \int_{x_i}^{x_f} m\frac{dv}{dt}dx = \int_{x_i}^{x_f} m\frac{dv}{dx}\frac{dx}{dt}dx = \int_{v_i}^{v_f} mvdv = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \qquad \text{.............(6)}$$

where v_i is the speed of the block at x_i and v_f is its speed at x_f.

Eq(6) tells us that the work done by the net force on a particle of mass m is equal to the difference between the initial and final values of a quantity $\frac{1}{2}mv^2$. This quantity is called the kinetic energy K

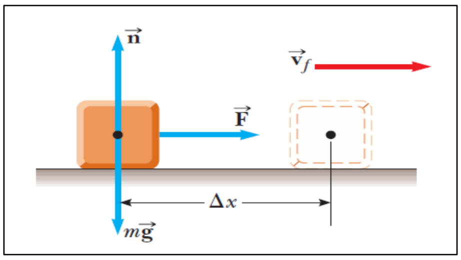
$$K = \frac{1}{2}mv^2 \qquad \text{.............(7)}$$

Kinetic energy is a scalar quantity and has the same units as work. It is often convenient to write eq(6) in the form

$$W = \Delta K \qquad \text{.............(8)}$$

Ex: A 6 kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3m.

Soln: $W = F\Delta x = (12N)(3m) = 36\ J$



$$W = K_f - K_i = \frac{1}{2}mv_f^2 - 0$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(36J)}{6Kg}} = 3.5 m/s$$

Ex: A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of $1x10^3$ N/m. The spring is compressed 2 cm and is then released from rest. Calculate the speed of the block as it passes through the equilibrium position x=0 if the surface is frictionless.

Soln:

$$W_s = \frac{1}{2}kx_{max}^2 = \frac{1}{2}(10^3)(-2\times10^{-2})^2 = 0.2\,J$$

$$W_s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad \rightarrow \quad 0.2 = \frac{1}{2}mv_f^2 - 0 = \frac{1}{2}(1.6)v_f^2 \quad \rightarrow \quad v_f = 0.5\,m/s$$

Ex: A particle is subject to a force F_x that varies with position as in figure. Find the work done by the force on the particle as it moves (a) from *x* =0 to *x*=5 m, (b) from *x*=5 m to *x*=10 m, and (c) from *x*=10 m to *x*=15 m. (d) What is the total work done by the force over the distance *x*=0 to *x*=15 m?

Soln:

(a) For the region $0 \le x \le 5.00$ m,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(b) For the region $5.00 \le x \le 10.0$,

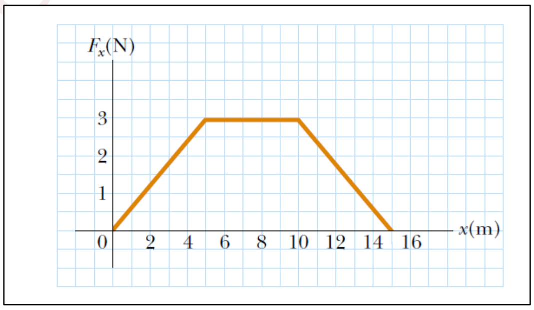
$$W = (3.00\text{ N})(5.00\text{ m}) = \boxed{15.0\text{ J}}$$

(c) For the region $10.0 \le x \le 15.0$,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(d) For the region $0 \le x \le 15.0$

$$W = (7.50 + 7.50 + 15.0)\text{ J} = \boxed{30.0\text{ J}}$$



Ex: A 3 kg object has a velocity $(6\hat{i} - 2\hat{j})$ m/s. (a) What is its kinetic energy at this time? (b) Find the total work done on the object if its velocity changes to $(8\hat{i} + 4\hat{j})$ m/s.

Soln:

$$v_i^2 = \vec{v}_i \cdot \vec{v}_i = 36 + 4 = 40\,m^2/s^2 \quad \rightarrow \quad v_i = \sqrt{40}\,m/s$$

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3)(40) = 60\,J$$

$$v_f^2 = \vec{v}_f \cdot \vec{v}_f = 64 + 16 = 80\,m^2/s^2 \quad \rightarrow \quad v_f = \sqrt{80}\,m/s$$

$$K_f = \frac{1}{2} m v_f^2 = \frac{1}{2}(3)(80) = 120\ \text{J}$$

$$W = \Delta K \rightarrow W = 120 - 60 = 60\ \text{J}$$

5-5- Potential Energy

Potential energy U is energy that can be associated with the configuration (arrangement) of a system of objects that exert forces on one another. The potential energy concept can be used only when dealing with a special class of forces called conservative forces. When only conservative forces act within an isolated system, the kinetic energy gained (or lost) by the system as its members change their relative positions is balanced by an equal loss (or gain) in potential energy.
The work done within the system by the conservative force equals the negative of the change in the potential energy ΔU of the system

$$\Delta U = -W$$

$$\Delta U = -\int_{x_i}^{x_f} F dx$$

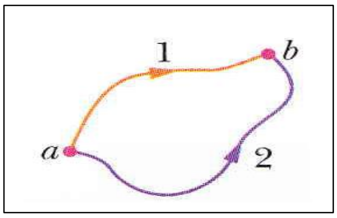
5-6- Conservative and non-conservative forces

Conservative forces have these two equivalent properties:
1- The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle. For example: gravitational force and spring force.

$W_{ab,1} = W_{ab,2}$

2- The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical.)

A force is non-conservative if it does not satisfy properties 1 and 2 for conservative forces. For example: frictional force.



A- Gravitational potential Energy

Consider a particle with mass m moving vertically along a y-axis. As the particle moves from point y_i to point y_f, the gravitational force does work on it. The change in the gravitational potential energy of the particle-Earth system

$$\Delta U = -\int_{y_i}^{y_f} F dy$$

$$\Delta U = -\int_{y_i}^{y_f} (-mg) dy$$

$$\Delta U = mgy_f - mgy_i = mg\Delta y$$

When the particle is at a reference point $y_i=0$ → $U_i=0$

$U = mgy$ (gravitational potential energy)

B- Elastic potential Energy

Consider the block-spring system, with the block moving on the end of a spring. As the block moves from point x_i to point x_f. The change in the elastic potential energy of the block-spring system,

$$\Delta U = -W$$

$$\Delta U = -\int_{x_i}^{x_f} F dx$$

$$\Delta U = -\int_{x_i}^{x_f} (-kx) dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

When the particle is at a reference point $x_i=0$ → $U_i=0$

$U = \frac{1}{2}kx^2$ (Elastic potential energy)

5-7- Conservation of mechanical energy

The mechanical energy E of a system is the sum of its potential energy U and the kinetic energy K of the objects within it:

$E = K + U$ (Mechanical energy)

When only conservative forces cause energy transfers within the system (when frictional forces do not act on the objects in the system). We shall assume that the system is isolated from its environment; that is, no external force from an object outside the system causes energy changes inside the system.

When a conservative force does work W on an object within the system, that force transfers energy between kinetic energy K of the object and potential energy U of the system.

The change ΔK in kinetic energy is

$W = \Delta K$ …………(1)

The change ΔU in potential energy is

$W = -\Delta U$ …………..(2)

Equate Eq(1) and (2)

$$K_f - K_i = -(U_f - U_i) \rightarrow K_f + U_f = K_i + U_i$$

$$E_f = E_i \rightarrow \Delta E = 0$$

This result is called the principle of conservation of mechanical energy for an isolated system.

Ex: A ball of mass m is dropped from a height h above the ground, as shown in figure below. (A) Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground. (B) What would its speed if the ball were thrown downward from its highest position with a speed v_i

Soln:

(A)

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2} m v_f^2 + mgy = 0 + mgh$$

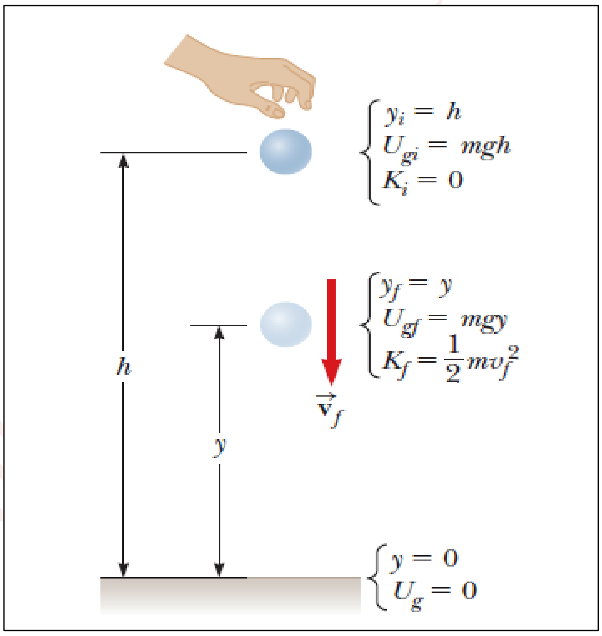
$$\frac{1}{2} m v_f^2 = mg(h-y) \rightarrow v_f = \sqrt{2g(h-y)}$$

(B)

$$\frac{1}{2} m v_f^2 + mgy = \frac{1}{2} m v_i^2 + mgh$$

$$\frac{1}{2} m v_f^2 = \frac{1}{2} m v_i^2 + mg(h-y)$$

$$v_f = \sqrt{v_i^2 + 2g(h-y)}$$



Ex: A pendulum consists of a sphere of mass m attached to a light cord of length L, as shown in the figure below. The sphere is released from rest at point Ⓐ when the cord makes an angle θ_A with the vertical, and the pivot at P is frictionless. (a) Find the speed of the sphere when it is at the lowest point Ⓑ. (b) What is the tension T_B in the cord at Ⓑ

Soln.

(a) Applying the principle of conservation of mechanical energy to the system gives:

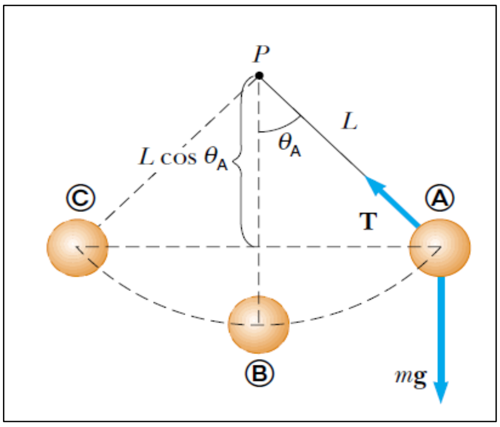
$$K_B + U_B = K_A + U_A$$

$$\frac{1}{2} m v_B^2 - mgL = 0 - mgL\cos\theta_A$$

$$\frac{1}{2} v_B^2 = gL(1 - \cos\theta_A)$$

$$v_B = \sqrt{2gL(1 - \cos\theta_A)} \qquad \ldots\ldots(1)$$

(b)



$$\sum F_r = ma_r \rightarrow T_B - mg = \frac{mv_B^2}{L}$$

$$T_B = m\left(g + \frac{v_B^2}{L}\right) \qquad \text{……(2)}$$

Substitute Eq.(1) into Eq.(2)

$$T_B = m\left(g + \frac{2gL(1-\cos\theta_A)}{L}\right) = mg(3 - 2\cos\theta_A)$$

5-8- Changes in mechanical energy for non-conservative forces

As we have seen, if the forces acting on objects within a system are conservative, then the mechanical energy of the system is conserved. However, if some of the forces acting on objects within the system are not conservative, then the mechanical energy of the system changes.

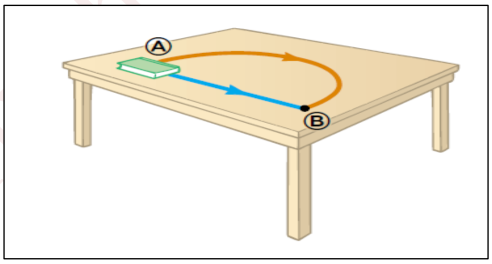
Consider the book sliding across the surface. As the book moves through a distance d, the only force that does work on it is the force of kinetic friction. This force causes a decrease in the kinetic energy of the book. This decrease leading to equation:

$$\Delta K = -f_k d$$

If the book moves on an incline that is not frictionless, there is a change in both the kinetic energy and the gravitational potential energy of the book–Earth system. Consequently,

$$\Delta K + \Delta U = -f_k d$$

$$\Delta E = -f_k d$$



Ex: A 3kg crate slides down a ramp. The ramp is 1m in length and inclined at an angle of 30°, as shown in figure below. The crate starts from rest at the top, experiences a constant friction force of magnitude 5N, and continues to move a short distance on the horizontal floor after it leaves the ramp. A) Use energy methods to determine the speed of the crate at the bottom of the ramp. B) How far does the crate slide on the horizontal floor if it continues to experience a friction force of magnitude 5 N?

Soln:

A)

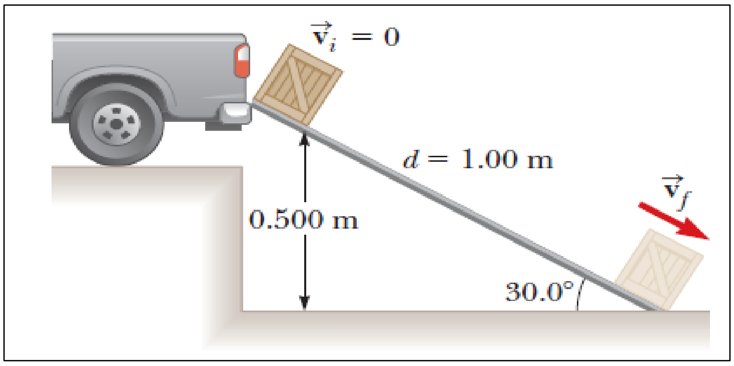
$$E_i = K_i + U_i$$

$$E_i = 0 + mgy_i = mgy_i$$

$$E_f = K_f + U_f$$

$$E_f = \frac{1}{2}mv_f^2 + 0 = \frac{1}{2}mv_f^2$$

$$\Delta E = -f_k d$$



$$\frac{1}{2}mv_f^2 - mgy_i = -f_k d$$

$$v_f = \sqrt{2gy_i - \frac{2f_k d}{m}} \rightarrow v_f = \sqrt{2(9.8)(0.5) - \frac{2(5)1}{3}} = 2.54\ \text{m/s}$$

B)

$$\Delta E = -f_k d$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -f_k d \rightarrow 0 - \frac{1}{2}mv_i^2 = -f_k d \rightarrow d = \frac{mv_i^2}{2f_k} = \frac{3(2.54)^2}{2(5)} = 1.94\ \text{m}$$

5-9- Relationship between conservative forces and potential energy

Let us imagine a system of particles in which the configuration changes due to the motion of one particle along the x axis. The work done by a conservative force F as a particle moves along the x axis is

$$\Delta U = -W$$

$$\Delta U = -\int_{x_i}^{x_f} F_x dx$$

$$dU = -F_x dx$$

The conservative force is related to the potential energy function through the relationship:

$$F_x = -\frac{dU}{dx} \quad \text{.........(1)}$$

We can easily check this relationship for the following two examples:

In the case of a spring , $U = \frac{1}{2}kx^2 \rightarrow F_x = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$

For gravitational potential energy function, $U = mgy \rightarrow F_y = -\frac{d}{dy}(mgy) = -mg$

Eq.(1), can be generalized in terms of three-dimension x, y, and z

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right)$$

The above Eq. can be rewritten as:

$$\vec{F} = -\vec{\nabla}U$$

Ex: A puck with coordinates x and y slides on a level, frictionless air hockey table. It is acted on by a conservative force described by the potential-energy function $U(x,y) = \frac{1}{2}k(x^2 + y^2)$, Find a vector expression for the force acting on the puck, and find an expression for the magnitude of the force.

Soln:

$$F_x = -\frac{\partial U}{\partial x} \rightarrow F_x = -\frac{\partial}{\partial x}\left(\frac{1}{2}k\left(x^2+y^2\right)\right) = -kx$$

$$F_y = -\frac{\partial U}{\partial y} \rightarrow F_y = -\frac{\partial}{\partial y}\left(\frac{1}{2}k\left(x^2+y^2\right)\right) = -ky$$

$$\vec{F} = -k\left(x\hat{i}+y\hat{j}\right) \rightarrow \left|\vec{F}\right| = k\sqrt{x^2+y^2}$$

Ex: A 6.0 kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3.0 m if the surfaces in contact have a coefficient of kinetic friction of 0.15.

Soln:

Method 1

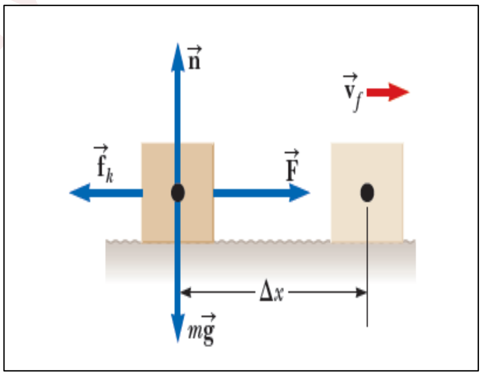
$$\sum F_y = 0 \rightarrow n - mg = 0 \rightarrow n = mg$$

$$W = \Delta K \rightarrow \left(\sum F_x\right)\Delta x = \Delta K \rightarrow (F - f_k)\Delta x = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$(F - f_k)\Delta x = \frac{1}{2}mv_f^2 - 0 \rightarrow (F - \mu_k n)\Delta x = \frac{1}{2}mv_f^2$$

$$2(F - \mu_k mg)\Delta x = mv_f^2$$

$$v_f = \sqrt{2\left(\frac{F}{m} - \mu_k g\right)\Delta x} = \sqrt{2\left(\frac{12}{6} - (0.15)(9.8)\right)3} = 1.78\ \text{m/s}$$



Method 2

$$\sum F_y = 0 \rightarrow n - mg = 0 \rightarrow n = mg$$

$$\sum F_x = ma_x \rightarrow F - f_k = ma_x \rightarrow F - mg\mu_k = ma_x \rightarrow a_x = \frac{F}{m} - g\mu_k$$

$$a_x = \frac{12}{6} - (9.8)(0.15) = 0.53\ \text{m/s}^2$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x\Delta x \rightarrow v_{xf}^2 = 0 + 2(0.53)(3) = 3.18\ \text{m}^2/\text{s}^2 \rightarrow v_{xf} = 1.78\ \text{m/s}$$

Ex: A child of mass m rides on an irregularly curved slide of height h=2.00 m, as shown in figure below. The child starts from rest at the top. (A) Determine his speed at the bottom, assuming no friction is present. (B) If a force of kinetic friction acts on the child, how much mechanical energy does the system lose? Assume that v_f = 3.00 m/s and m=20.0 kg.

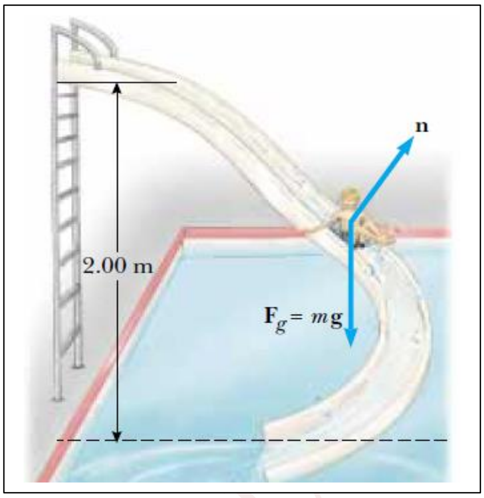
Soln:

A)

$K_f + U_f = K_i + U_i$

$\frac{1}{2}mv_f^2 + 0 = 0 + mgh \rightarrow$

$v_f = \sqrt{2gh} \rightarrow v_f = \sqrt{2(9.8)(2)} = 6.26 m/s$



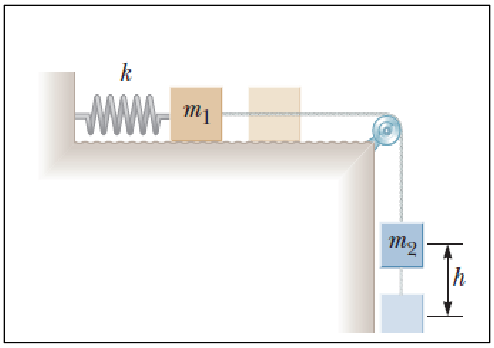
B)

$\Delta E = (K_f + U_f) - (K_i + U_i)$

$\Delta E = \left(\frac{1}{2}mv_f^2 + 0\right) - (0 + mgh) = \frac{1}{2}mv_f^2 - mgh$

$\Delta E = \left(\frac{1}{2}(20)(3)^2 + 0\right) - ((20)(9.8)(2)) = -302J$

Ex: Two blocks are connected by a light string that passes over a frictionless pulley as shown in figure below. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant *k.* The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance *h* before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.



Soln

$\sum F_y = 0 \rightarrow n - m_1 g = 0 \rightarrow n = m_1 g$

$\Delta E = -f_k d$

$(K_{f1} + U_{gf1} + K_{f2} + U_{gf2} + U_{sf}) - (K_{i1} + U_{gi1} + K_{i2} + U_{gi2} + U_{si}) = -f_k d$

$\left(0 + 0 + 0 - m_2 gh + \frac{1}{2}kh^2\right) - (0 + 0 + 0 + 0 + 0) = -f_k d$

$-m_2 gh + \frac{1}{2}kh^2 = -f_k d \rightarrow -m_2 gh + \frac{1}{2}kh^2 = -m_1 g \mu_k h$

$$\mu_k = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$$