Chapter Two
Motion in one dimension

2.1 Position

A particle's **position** x is the location of the particle with respect to a chosen reference point that we can consider to be the origin of a coordinate system.

2.2 Displacement and Distance

The **displacement** $\boldsymbol{\Delta x}$ of a particle is defined as its change in position. As it moves from an initial position $\boldsymbol{x_i}$ to a final position $\boldsymbol{x_f}$, we write the displacement of the particle as

$$\boldsymbol{\Delta x = x_f - x_i}$$

From this definition we see that $\boldsymbol{\Delta x}$ is positive if $\boldsymbol{x_f}$ is greater than $\boldsymbol{x_i}$ and negative if $\boldsymbol{x_f}$ is less than $\boldsymbol{x_i}$.

Displacement is an example of a vector quantity. We use plus and minus signs to indicate vector direction. Any object always moving to the right undergoes a positive displacement $\boldsymbol{+\Delta x}$, and any object moving to the left undergoes a negative displacement $\boldsymbol{-\Delta x}$.It is very important to recognize the difference between displacement and distance travelled. **Distance d** is the length of a path followed by a particle.

Ex: What is the difference between distance and displacement?

Displacement	Distance
Vector quantity (has direction and magnitude)	Scalar quantity (has magnitude only)
Positive or negative	Always positive
It's magnitude is shortest length between two points	is the length between two points longer than straight line between them

2.3 Average velocity and speed

The **average velocity** $\bar{v}_x$ of a particle is defined as the particle's displacement Δx divided by the time interval Δt during which that displacement occurred:

$$\bar{v}_x = \frac{\Delta x}{\Delta t}$$

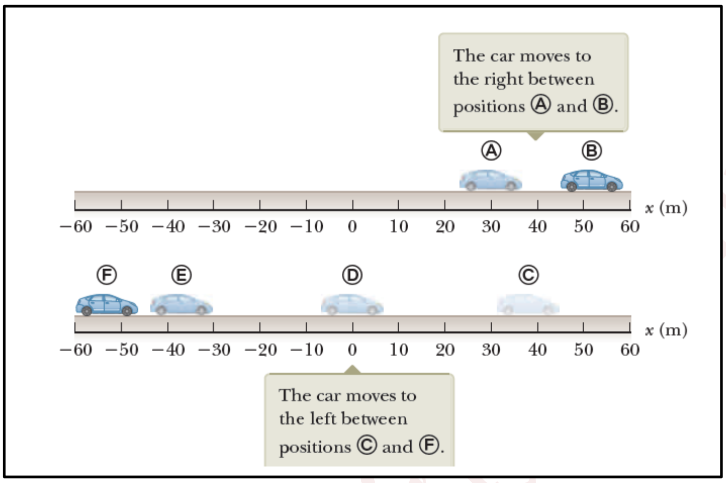
where the subscript "x" indicates motion along the x-axis. The average velocity has dimensions of length divided by time (L/T), or meters per second in SI units.

There is a clear distinction between speed and velocity. The **average speed** $\bar{v}$ of a particle, a scalar quantity, is defined as the total distance travelled **d** divided by the total time it takes to travel that distance:

$$\bar{v} = \frac{d}{\Delta t}$$

The SI unit of average speed is the same as the unit of average velocity: meters per second. However, unlike average velocity, the average speed has no direction and hence carries no algebraic sign.

Ex: Find the displacement, average velocity, and average speed of the car in the following figure between positions A and F. Note that X_A=30 m at t= 0 sec and that X_F = -53m at t=50 sec



Position	t(s)	x(m)
A	0	30
B	10	52
C	20	38
D	30	0
E	40	-37
F	50	-53

Solution

$$\Delta x = x_F - x_A = -53\text{m} - 30\text{m} = -83\text{m}$$

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{-53-30}{50-0} = -1.7 \text{ m/s}$$

$$\bar{v} = \frac{d}{\Delta t} = \frac{127}{50} = 2.5\text{m/s}$$

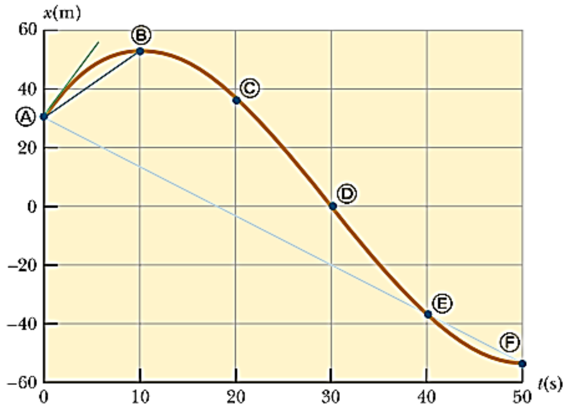
2.4 Instantaneous velocity and speed

Instantaneous velocity v_x equals the limiting value of the ratio $\Delta x/\Delta t$ as Δt approaches zero

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The instantaneous velocity can be positive, negative, or zero. In the following figure, when the slope of the position-time graph is positive, such as at any time during the first 10s, v_x is positive. After point B, v_x is negative because the slope is negative.

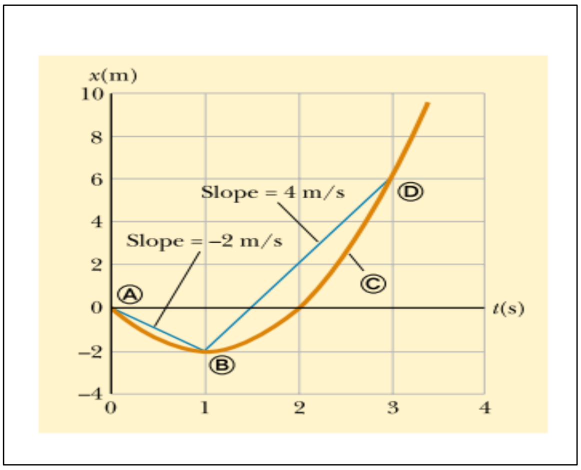
At the peak, the slope and the instantaneous velocity are zero. The instantaneous speed of a particle is defined as the magnitude of its velocity.



Ex: A particle moves along the x-axis. Its x coordinate varies with time according to the expression $x=-4t+2t^2$ where x is in meters and t is in seconds. The position-time graph for this motion is shown in the following figure. Note that the particle moves in the negative x direction for the first second of motion, is at rest at the moment t =1 s, and moves in the positive x direction for t>1 s. a) Determine the displacement of the particle in the time intervals t=0 to t=1 sec and t=1 s to t=3 s. b) Calculate the average velocity during these two time intervals. c) Find the instantaneous velocity of the particle at t=2.5 s.

Solution

a)



$$\Delta x_{Ⓐ\to Ⓑ} = x_f - x_i = x_Ⓑ - x_Ⓐ$$
$$= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2] = -2 \text{ m}$$

$$\Delta x_{Ⓑ\to Ⓓ} = x_f - x_i = x_Ⓓ - x_Ⓑ$$
$$= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2] = +8 \text{ m}$$

b)

$$\bar{v}_{x\,(Ⓐ\to Ⓑ)} = \frac{\Delta x_{Ⓐ\to Ⓑ}}{\Delta t} = \frac{-2 \text{ m}}{1 \text{ s}} = -2 \text{ m/s}$$

$$\bar{v}_{x\,(Ⓑ\to Ⓓ)} = \frac{\Delta x_{Ⓑ\to Ⓓ}}{\Delta t} = \frac{8 \text{ m}}{2 \text{ s}} = +4 \text{ m/s}$$

c)

$$v_x = \frac{dx}{dt} = -4 + 4t$$

$$v_x(t = 2.5) = 6m/s$$

2. 5 Acceleration

The **average acceleration** $\bar{a}_x$of the particle is defined as the change in velocity Δv_x divided by the time interval Δt during which that change occurred:

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{x_f} - v_{x_i}}{t_f - t_i}$$

As with velocity, we can use positive and negative signs to indicate the direction of the acceleration. Acceleration has dimensions of length divided by time squared, or L/T^2. The SI unit of acceleration is meters per second squared (m/sec^2). The **instantaneous acceleration** equals the derivative of the velocity with respect to time

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

If a_x is positive, then the acceleration is in the positive x direction; if a_x is negative, then the acceleration is in the negative x direction. The acceleration can also be written

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Ex: The velocity of a particle moving along the x-axis varies in time according to the expression v_x=(40-5t^2) m/s, where t is in seconds. (a) Find the average acceleration in the time interval t=0 to t= 2.0 s. (b) Determine the acceleration at t = 2.0 s.

Solution

a)

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

$v_{xA}=40-5(0)^2=40$ m/s

$v_{xB}=40-5(2)^2=20$ m/s

$$\bar{a}_x = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{(20-40)m/s}{(2-0)s} = -10\ \mathrm{m/s^2}$$

b)

$$a_x = \frac{dv_x}{dt} = -10t\ \mathrm{m/s^2} \quad \rightarrow \quad a_x(t=2s) = -20\ \mathrm{m/s^2}$$

2.6 One-dimensional motion with constant acceleration

If the acceleration of a particle varies in time, its motion can be complex and difficult to analyze. However, a very common and simple type of one-dimensional motion is that in which the acceleration is constant.

If the acceleration is constant, then the average acceleration is equal to instantaneous acceleration, i.e

$$\bar{a}_x = a_x$$

If we take t_i=0 and t_f to be any later time t, we find that

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$

$v_{xf} = v_{xi} + a_x t$ (for constant a_x) --- ------ (1)

A velocity-time graph for this constant acceleration motion is shown in the following Figure 2.1(a). When the acceleration is constant, the graph of acceleration versus time Figure 2.1(b) is a straight line having a slope of zero.

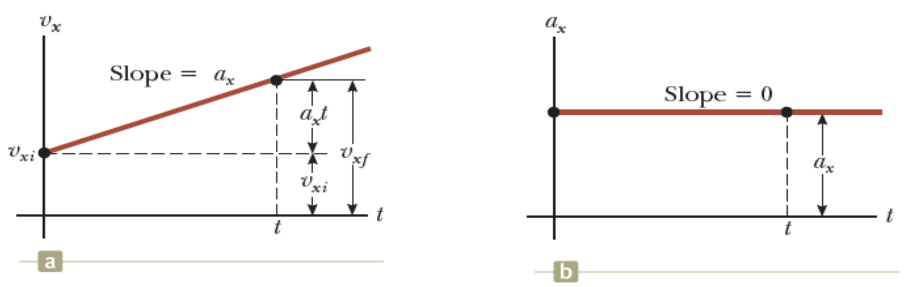


Fig2.1: A particle under constant acceleration a_x moving along the x-axis:
(a) The velocity–time graph, and (b) the acceleration– time graph.

Because velocity at constant acceleration varies linearly in time, we can express the average velocity in any time interval as

$\bar{v}_x = \frac{v_{xf} + v_{xi}}{2}$ (for constant a_x) ----------------- (2)

Recalling that $\bar{v}_x = \frac{x_f - x_i}{t_f - t_i} = \frac{x_f - x_i}{t}$

Equating the above equation with equation (2), we get:

$x_f - x_i = \frac{v_{xf} + v_{xi}}{2} t$ (for constant a_x) ------------ (3)

We can obtain another useful expression for displacement at constant acceleration by substituting equation (1) into equation (3): $x_f - x_i = \frac{1}{2}(2v_{xi} + a_x t)t$

$x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$ (for constant a_x) ------------- (4)

Finally, we can obtain an expression for the final velocity that does not contain a time interval by substituting the value of “t” from equation (1) into equation (3):

$$x_f - x_i = \left(\frac{v_{xf} + v_{xi}}{2}\right)\left(\frac{v_{xf} - v_{xi}}{a_x}\right)$$

$x_f - x_i = \left(\frac{v_{xf}^2 - v_{xi}^2}{2a_x}\right)$

$v_{xf}^2 = v_{xi}^2 + 2a_x\Delta x$ (for constant a_x) - - - -- - - - (5)

Ex: A jet lands on an aircraft carrier at 63 m/s. **(a)** What is its acceleration if it stops in 2 s? **(b)** What is the displacement of the plane while it is stopping?

Solution

(a) $a_x = \frac{v_{xf} - v_{xi}}{t} = \frac{0-63}{2} = -31.5\ \mathrm{m/s^2}$

(b) $x_f - x_i = \frac{v_{xf} + v_{xi}}{2}t = \left(\frac{0+63}{2}\right)(2) = 63m$

Ex: A car traveling at a constant speed of 45.0 m/s passes a trooper on a motorcycle hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch the car, accelerating at a constant rate of 3.0 m/s^2. How long does it take the trooper to overtake the car?

Solution

Choose the position of the billboard as the origin x_B =0

For car $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$ → x_{car} = 45m+(45 m/s)t+0 = 45+45t

For trooper $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$ → $x_{trooper} = 0 + (0)t + \frac{1}{2}(3)t^2 = 1.5t^2$

The trooper overtaking the car when $x_{trooper} = x_{car}$

$$45 + 45\,t = 1.5t^2$$

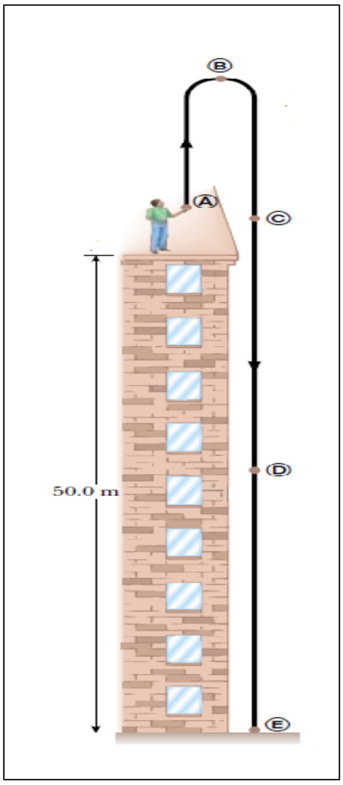
This gives the quadratic equation $1.5t^2 - 45t - 45 = 0$

$$t = \frac{45 + \sqrt{45^2 + 4(1.5)(45)}}{2(1.5)} = 31\ \text{s.}$$

2.7 Freely falling objects

In the absence of air resistance, all objects dropped near the Earth's surface fall toward the Earth with the same constant acceleration under the influence of the Earth's gravity. **A freely falling object** is any object moving freely under the influence of gravity alone, regardless of its initial motion. Objects were thrown upward or downward and those released from rest are all falling freely once they are released. Any freely falling object experiences acceleration directed downward, regardless of its initial motion. It is common to define "up" as the +y direction and to use y as the position variable in the kinematic equations. At the Earth's surface, the value of g is approximately 9.80 m/s^2. The equations developed above for objects moving with constant acceleration can be applied. The only modification that we need to make in these equations for freely falling objects is to note that the motion is in the vertical direction (the y direction) rather than in the horizontal (x) direction and that the acceleration is downward and has a magnitude of 9.80 m/s^2. Thus, we always take a_y=-g=-9.8 m/s^2, where the minus sign means that the acceleration of a freely falling object is downward.

Ex: A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50 m high, and the stone just misses the edge of the roof on its way down. Using t_A=0 as the time the stone leaves the thrower's hand at position A, determine **(a)** the time at which the stone reaches its maximum height, **(b)** the maximum height, **(c)** the time at which the stone returns to the height from which it was thrown, **(d)** the velocity of the stone at this instant, and **(e)** the velocity and position of the stone at t = 5 s.



Solution.

a)

$$v_{yf} = v_{yi} + a_y t \quad \rightarrow \quad t = \frac{v_{yf} - v_{yi}}{a_y} \quad \rightarrow \quad t_B = \frac{0-20}{-9.8} = 2.04 \text{ s}$$

b) $$y_{max} = y_B = y_A + v_{yA} t + \frac{1}{2} a_y t^2$$

$$y_B = 0 + (20\text{m/s})(2.04\text{m}) + \frac{1}{2}(-9.80\text{m/s})(2.04\text{s})^2 = 20.4\text{m}$$

c)

$$y_c - y_A = v_{yA} t + \frac{1}{2} a_y t^2 \quad \rightarrow \quad 0 - 0 = 20t - \frac{1}{2}(9.8)t^2$$

$$t(20 - 4.9t) = 0 \quad \rightarrow \quad t = 4.08 \text{ sec}$$

d)

$v_{yf}^2 = v_{yi}^2 + 2a_y \Delta y$

$v_{yC}^2 = v_{yA}^2 + 2a_y(y_C - y_A)$

$v_{yC}^2 = (20)^2 + 2(9.8) - (0 - 0) = 400\ \text{m}^2/s^2$

$v_{yC} = -20\ \text{m}/s$

e)

$v_{yf} = v_{yi} + a_y t$

$v_{yD} = v_{yA} + a_y t$

$v_{yD} = 20 + (-9.8)5 = -29\ \text{m/s}$

$y_D = y_A + v_{yA} t + \frac{1}{2} a_y t^2$

$y_D = 0 + (20)(5) + \frac{1}{2}(-9.8)(5)^2 = -22.5\ \text{m}$