Logic design

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First stage

(Lecture 4)

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Chapter one

-Number systems and codes-

<u>Complements :</u>

Complements are used in digital computer for simplifying the subtraction operation and for logical manipulation. There are two types of complement for each base (R) system:

1-The R´s complement
2-the (R-1)´s complement

For binary number → 1´s and 2´s complement
For decimal number → 9´s and 10´s complement
For octal number → 7's and 8's complement
For hexadecimal number → 15's and 16's complement

<u>The 1´s and 2´s complement :</u>

The 1´s complement of a binary number is the no. we get when we change each (0) to (1) and each (1) to (0) (or subtracting each binary no. from 1)

EX: 1´s comp. of 1001 → 0110
1´s comp. of 110010 → 001101
2´s comp. = 1´s comp. + 1
2´s comp. of 1011 is 0100 + 1 = 0101
2´s comp. of 1110 is 0001 + 1= 0010

Using 1´s complement in subtraction:

1’s complement subtraction is a method to subtract two binary numbers. This method allows subtraction of two binary numbers by addition. The 1’s complement of a binary number can be obtained by replacing all 0 to 1 and all 1 to 0. This article discusses steps involved in 1’s complement subtraction of a smaller number from a larger number & the steps for subtracting a smaller number from a larger number with example.

Steps for 1’s Complement Subtraction:

Subtraction of Smaller Number from Larger Number:

The steps for 1’s complement subtraction of a smaller number from a larger binary number are as follows:

Step-1: Determine the 1’s complement of the smaller number.

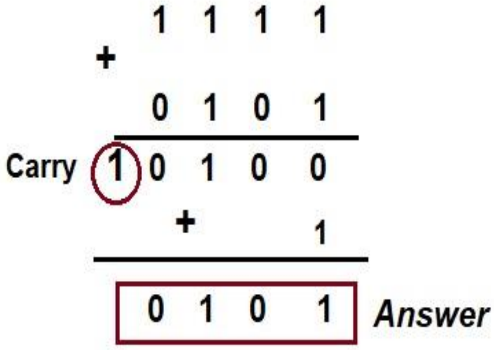
Step-2: Add this to the larger number.

Step-3: Remove the carry and add it to the result. This carry is called end-around-carry.

Example-1: Subtract $(1010)_2$ from $(1111)_2$ using 1’s complement method.

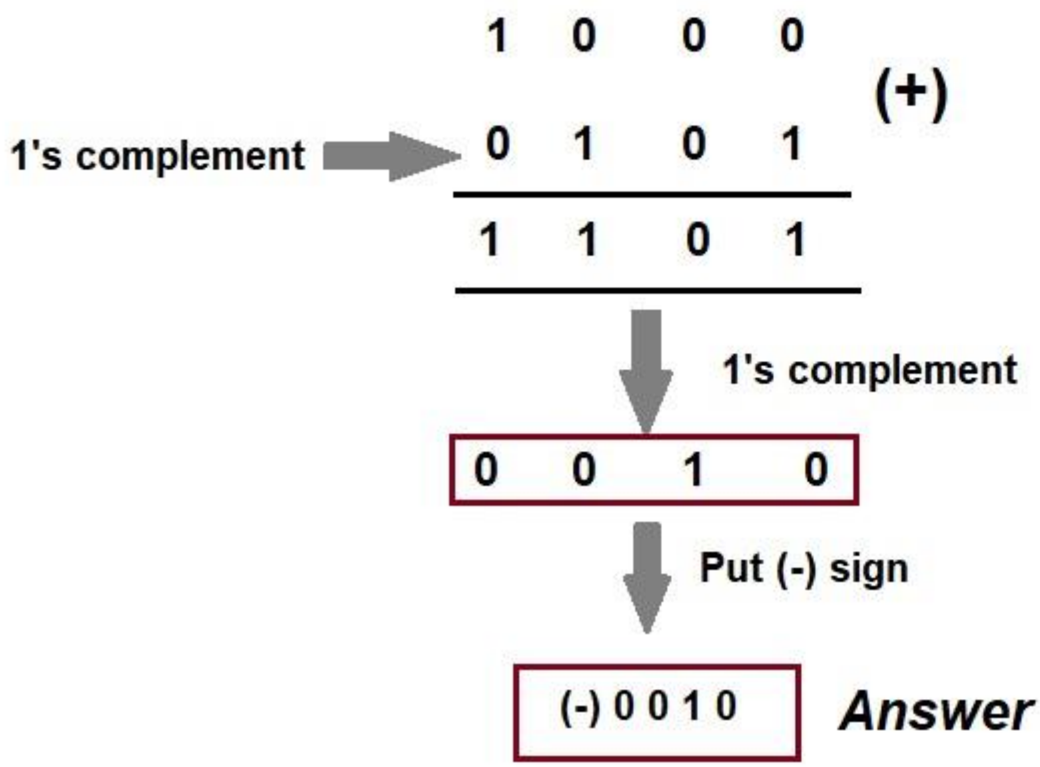
Step-1: Find the 1’s complement of 1010. It will be found by replacing all 0 to 1 and all 1 to 0. In this way, the required 1’s complement will be 0101.

Step-2: In this step, we need to add the vale calculated in step-1 to 1111. This is shown below.



Example: Subtract $(1010)_2$ from $(1000)_2$ using 1's complement method.

The 1's complement of $(1010)_2$ is $(0101)_2$. Now, we will add this with the smaller number and finally take 1's complement of the result to get the answer. This is shown below.



Mind that, no carry has been obtained while subtracting a larger number from a smaller number. Further, a minus sign has been put.

Using 2´s complement in subtraction:

The 2's complement is a way of representing signed integers in binary form. It is generally used in computer systems and digital arithmetic operations. In the 2's complement representation, the MSB (most significant bit) is used to indicate the sign of the number. 0 represents a positive number, and 1 illustrates a negative number.

Subtracting binary numbers using 2's complement involves converting the subtrahend (the number being subtracted) into its 2's complement form. Then add it to the minuend (the number from which the subtrahend is being subtracted). If there is a **carry-out** of the most significant bit, it indicates that the result is **negative**.

=> X - Y

=> X + (2’s complement of Y)

Here are the steps to subtract binary numbers using 2's complement:

1. Convert the subtrahend into its 2's complement form by inverting all the bits and adding 1.

2. Add the minuend to the 2's complement form of the subtrahend, ignoring any carry from the previous addition.

3. If there is a carry-out of the most significant bit (leftmost bit), discard it. This indicates that the result is negative.

4. The resulting binary number is the difference between the minuend and subtrahend in binary form.

Sample Example

Here's an example of subtracting two binary numbers using 2's complement:
Minuend: 1101
Subtrahend: 1010

1. Convert the subtrahend into its 2's complement form by inverting all the bits and adding 1:

1010 -> 0101 + 1 = 0110
2. Add the minuend to the 2's complement form of the subtrahend:

3. Ignore any carry from the previous addition:
0011

4. The resulting binary number is the difference between the minuend and subtrahend in binary form:
0011 = 3 in decimal

Therefore, **1101 - 1010 = 3 in decimal using 2's complement subtraction.**

Instead of subtraction a number , we can add it's 2's comp, and disregard the last carry.

EX: decimal

7 111 → 111

-5 -101 1´s → 010 2´s → 011

2 1+ 1 010 +

ve. No.

011 X carry

EX: 13 1101 → 1101

-10 1010 1´s → 0101 2´s → 0110

3 1+ 1 0011

+ve. No.

0110 X carry

EX: 4 100 → 100

-7 -111 1´s → 000 2´s → 001+

-3 1+ 101

001 No carry →

-ve. No.

So 101 → 100 → 011

There are three ways in which signed binary numbers may be expressed:

- Signed magnitude (SM)
- One's complement and
- Two's complement.

In an 8-bit word, signed magnitude representation places the absolute value of the number in the 7 bits to the right of the sign bit.

Ex: in **8-bit signed magnitude(SM),** positive 3 is: 00000011

Negative 3 is: 10000011

Ex: in 8-bit **one's complement**, positive 3 is: 00000011

Negative 3 is: 11111100

Ex: Adding 1 gives us -3 in **two's complement** form: 11111101

Ex: convert using SM method $\mathbf{(01011001)_2}$ = $+(1 * 2^6 + 0 * 2^5 + 1 * 2^4 + 1 * 2^3 + 0 * 2^2 + 0 * 2^1 + 1 * 2^0)$

= + (64 + 0 + 16 + 8 + 0 + 0 + 1)

= $\mathbf{(+89)_{10}}$

Ex: convert using SM method $\mathbf{(10011100)_2}$ = $-(0 * 2^6 + 0 * 2^5 + 1 * 2^4 + 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 0 * 2^0)$

= - (0 + 0 + 16 + 8 + 4 + 0 + 0)

= $\mathbf{(-28)_{10}}$

Homework:-

Find by using 1's and 2's complement

1- 10 – 7
2- 18-22
3- 44-11
4- 30-15
5- 20-51