Chapter -2-

Solving of non-linear equations (finding of the roots)

General notes :-

- Our equations must be written in the form

$$f(x) = 0$$

Example:

$$x^2 - 3x + 2 = 0$$

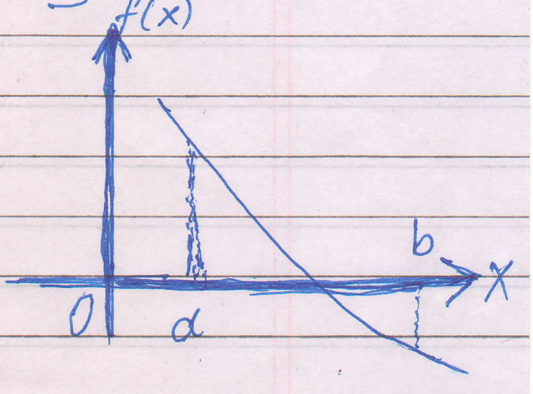
$$xe^x - 4 = \ln x$$

$$\Rightarrow xe^x - \ln x - 4 = 0$$

- $f(x)$ is any function of the variable (x).
- The root of the equation $f(x)=0$ is the value of (x) which satisfies the equation or the root is the value of (x) which makes $f(x)$ equal to Zero.
- We shall denote the root by $(\bar{x})$.
- The equation $f(x)=0$ may have more than One root.
- We shall take some iterative numerical methods for root finding.
- In many of these methods, we need to find the interval which contains the root (the root maybe positive or negative).
- For a positive root (+ve), we make the table

x	0	0.5	1	1.5	2
f(x)	+	+	+	−	−

a b



there is a root in the interval [a,b].

- For a negative root we take (-ve) values for (x) (starting from Zero if possible).
- In iterative methods, we get closer and closer to the real root by calculating many values of (x).
- Each calculated x is known as an instanteneous root : $x_1, x_2, x_3, \ldots$ (in general x_i) (i may take 0-value).
- The more close x_i to the real root, the more accurate solution.
- We may use the notations : x_i, x_{i+1}, x_{i-1}. for example, if $i=3$:

$$x_i = x_3 \,,\; x_{i+1} = x_4 \,,\; x_{i-1} = x_2$$

- When the instantaneous root (x_i) gets closer to the real root, the function $f(x_i)$ gets closer to Zero. Or when $x_i \longrightarrow \bar{x}$ then $f(x_i) \longrightarrow 0$.
- In general, full accuracy is not obtained in numerical methods and we may consider the root as that value of (x_i) which makes $|f(x_i)| \leq \epsilon$, where ϵ is a small quantity.
- For smaller ϵ we get higher accuracy.
- Accuracy maybe represented by different stopping conditions :-

1- Absolute error :

$$|x_{i+1} - x_i| \leq \epsilon$$

2- Relative error: $\left|\frac{x_{i+1}-x_i}{x_{i+1}}\right| \leq \epsilon$

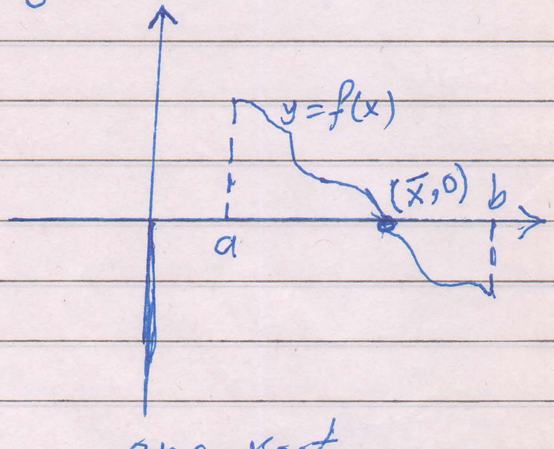
3- Percentage error: $\left|\frac{x_{i+1}-x_i}{x_{i+1}}\right| * 100 \leq \epsilon$.

4. $|f(x_i)| \leq \epsilon$ or $|f(x_{i+1})| \leq \epsilon$.

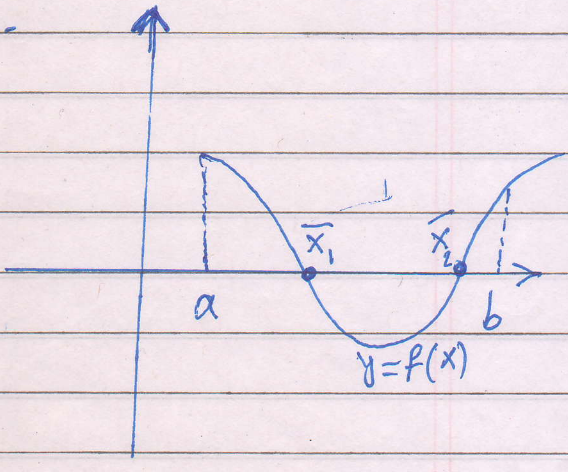
There are two methods to find the initial value of the root for the equations :-

① Graphical method :

The roots of the equation $f(x)=0$ are the intersection points of the curve of the function $y=f(x)$ with the x-axis.



one root

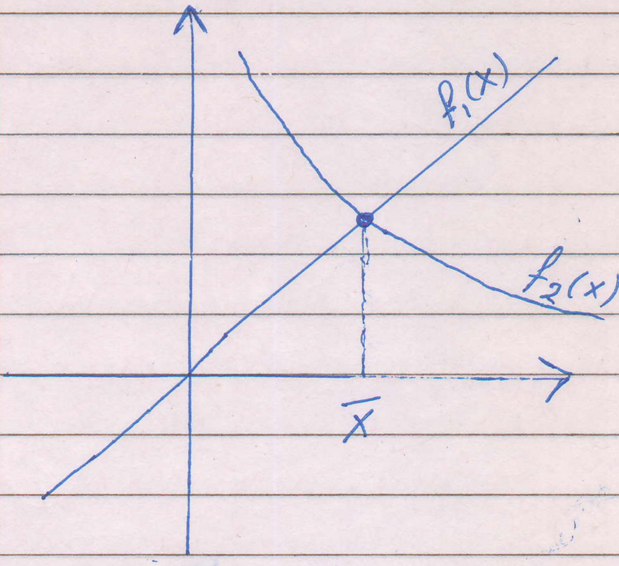


two roots

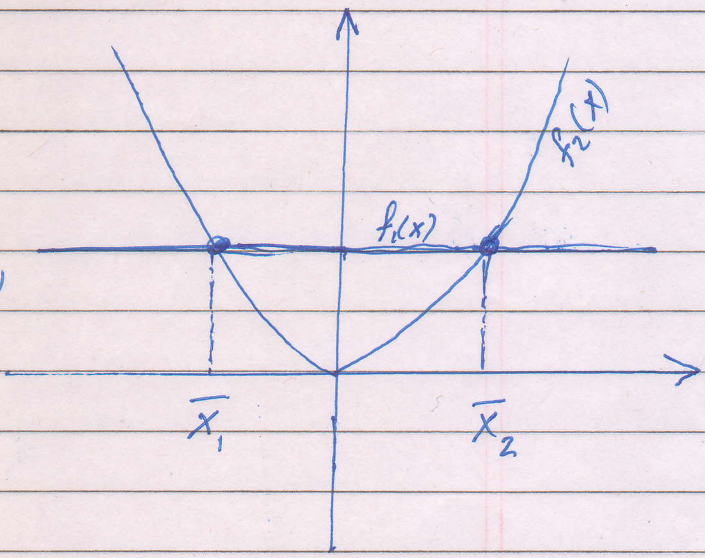
In some cases it is better to write an equation $f(x)=0$ in the form:

$$f_1(x) = f_2(x)$$

after that we draw two functions $y=f_1(x)$ and $y=f_2(x)$. If the intersection points $(\bar{x},\bar{y})$ of the curves then $\bar{x}$ represents a root of the equation



One root



two roots

Example:

Find the solutions of the following equations by the graphical method:

① $x^2-1=0$, ② $xe^x=1$

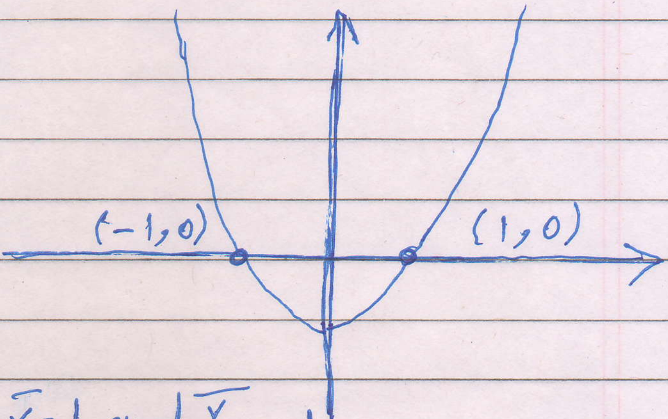
Solution:

① $x^2-1=0$

Case 1

$y=x^2-1$

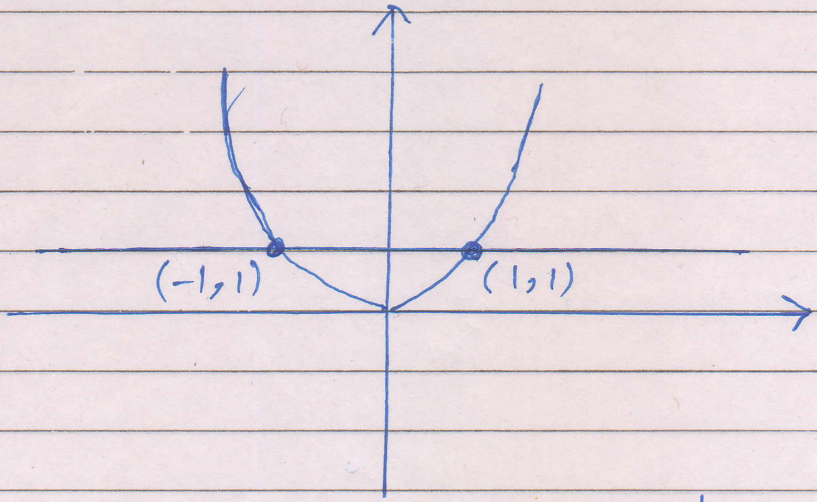
x	y
0	-1
-1	0
1	0



∴ the roots are $\bar{x}_1=1$ and $\bar{x}_2=-1$

Case 2

$x^2-1=0 \Rightarrow x^2=1 \Rightarrow f_1(x)=x^2$ and $f_2(x)=1$



∴ The roots are $\bar{x}_1=1$ and $\bar{x}_2=-1$.

② $xe^x=1$

$xe^x=1 \Rightarrow e^x=\frac{1}{x} \Rightarrow f_1(x)=e^x$ and $f_2(x)=\frac{1}{x}$