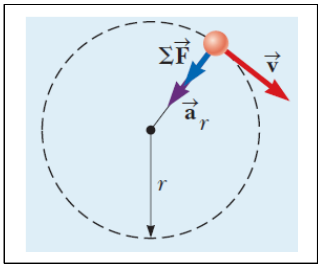
4-9- Centripetal Force

According to Newton's second law, if acceleration occurs, a net force must be causing it. Therefore, when a particle travels in a circular path, a force must be acting inward on the particle. The net force acting on the particle along the radial direction is called centripetal force given by:

$$\sum F_r = ma_r = m\frac{v^2}{r}$$



Ex: A puck of mass 0.5 kg is attached to the end of a cord 1.5 m long. The puck moves in a horizontal circle. If the cord can withstand a maximum tension of 50 N, what is the maximum speed at which the puck can move before the cord breaks? Assume the string remains horizontal during the motion.

Soln:

$$\sum F_r = ma_r \rightarrow T = m\frac{v^2}{r} \rightarrow v = \sqrt{\frac{Tr}{m}} \rightarrow v_{max} = \sqrt{\frac{T_{max}r}{m}} \rightarrow v_{max} = \sqrt{\frac{(50)(1.5)}{0.50}} = 12.2\ \text{m/s}$$

Ex: Estimate the force a person must exert on a string attached to a 0.15 kg ball to make the ball revolve in a horizontal circle of radius 0.6 m, where the ball makes 30 revolutions every 10 sec.

Soln:

Apply Newton's second law to the radial direction

$$\sum F_r = ma_r = m\frac{v^2}{r} = m\frac{4\pi^2 r}{T^2} = 0.15\times\left[\frac{4\pi^2(0.6)}{(10/30)^2}\right] = 40\text{N}$$

Ex: A pilot is flying a small plane at 30 m/s in a circular path with a radius of 100 m. If a force of 635 N is needed to maintain the pilot's circular motion, what is the pilot's mass?

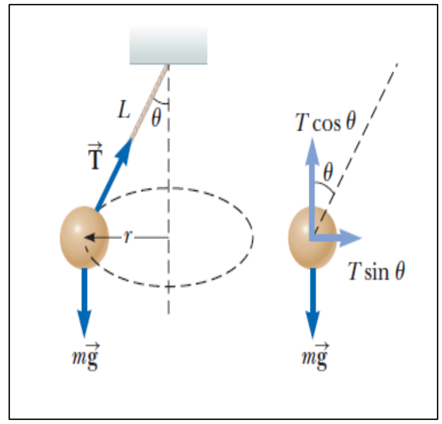
Soln:

Apply Newton's second law

$$\sum F_r = m\frac{v^2}{r} \rightarrow 635 = m\frac{(30)^2}{100} \rightarrow m = \frac{63500}{900} = 71\,\text{Kg}$$

Ex: (*Conical pendulum*) A small ball of mass *m* is suspended from a string of length *L*. The ball revolves with constant speed in a horizontal circle of radius *r* as shown in figure below. Find expressions for the tension, speed and the period when the string make an angle θ with the vertical.

Soln:
Let assume T is the tension in the string and t is the period.



$$\sum F_x = T\sin\theta = \frac{mv^2}{r} \rightarrow T\sin\theta = \frac{mv^2}{r} \quad \ldots..(1)$$

$$\sum F_y = T\cos\theta - mg = 0 \rightarrow T\cos\theta = mg \quad \ldots..(2)$$

Divided equation (1) on (2)

$$\tan\theta = \frac{v^2}{rg} \rightarrow v = \sqrt{rg\tan\theta}$$

$$r = L\sin\theta \rightarrow v = \sqrt{Lg\sin\theta\tan\theta}$$

$$t = \frac{2\pi r}{v} = \frac{2\pi L\sin\theta}{\sqrt{Lg\sin\theta\tan\theta}} = 2\pi\sqrt{\frac{L}{g}\cos\theta}$$

Ex: A sports car is rounding a flat, unbanked curve with radius 35 m. If the coefficient of static friction between tires and road is 0.523 what is the maximum speed at which the driver can take the curve without sliding.

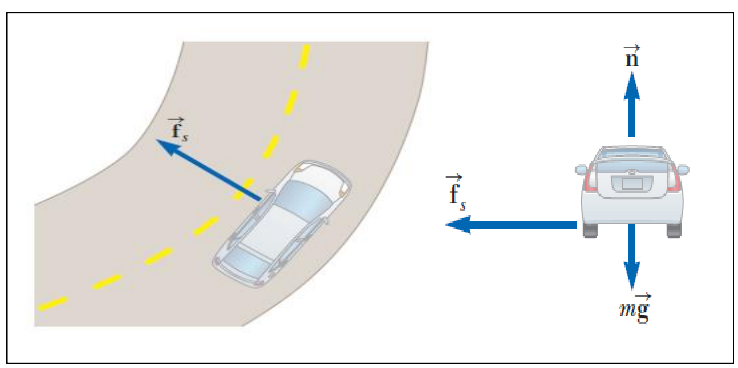
Soln:

$$\sum F_r = ma_r = m\frac{v^2}{r} \rightarrow f_{s.max} = \mu_s n = m\frac{v^2}{r}$$

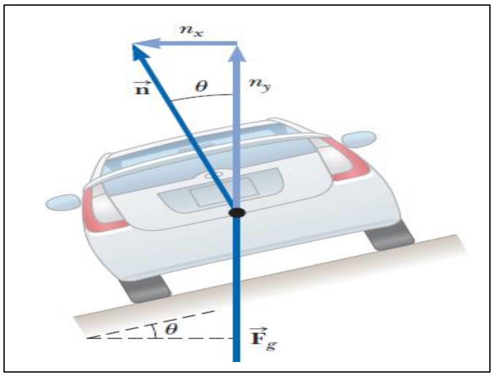
$$\sum F_y = 0 \rightarrow n - mg = 0 \rightarrow n = mg$$

$$v_{max} = \sqrt{\frac{\mu_s nr}{m}} = \sqrt{\frac{\mu_s mgr}{m}} = \sqrt{\mu_s gr}$$

$$v_{max} = \sqrt{(0.523)(9.8)(35)} = 13.4 \ m/s$$



Ex: A civil engineer wishes to redesign a curved roadway in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a road is usually banked, which means that the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be 13.4 m/s and the radius of the curve is 35 m. At what angle should the curve be banked?



Soln:

$$\sum F_r = ma_r = m\frac{v^2}{r} \rightarrow n\sin\theta = \frac{mv^2}{r} \quad ....(1)$$

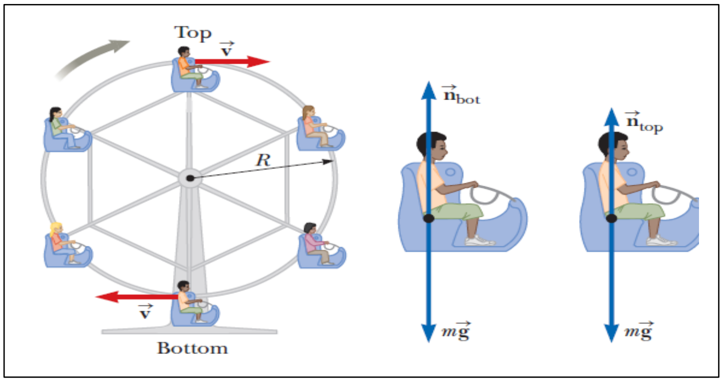
$$\sum F_y = n\cos\theta - mg = 0 \rightarrow n\cos\theta = mg \quad ....(2)$$

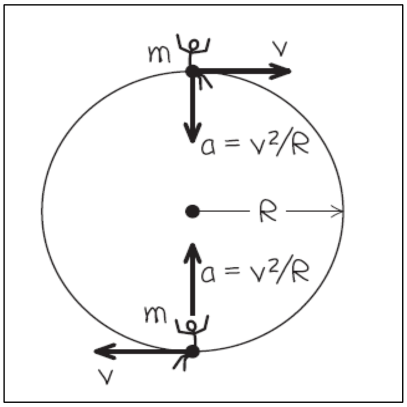
Divided equation (1) on (2)

$$\tan\theta = \frac{v^2}{rg} \rightarrow \theta = \tan^{-1}\left[\frac{(13.4)^2}{(35)(9.8)}\right] = 27.6^0$$

Ex: A child of mass m rides on a Ferris wheel as shown in figure below. The child moves in a vertical circle of radius 10 m at a constant speed of 3 m/s. (A) determine the force exerted by the seat on the child at the bottom of the ride. (B) Determine the force exerted by the seat on the child at the top of the ride. Express your answer in terms of the weight of the child, mg.

Soln:





(A) $$\sum F_y = n_{bot} - mg = m\frac{v^2}{r}$$

$$n_{bot} = mg + m\frac{v^2}{r} = mg\left(1 + \frac{v^2}{rg}\right)$$

$$n_{bot} = mg\left(1 + \frac{3^2}{(10)(9.8)}\right) = 1.09\,mg$$

(B) $$\sum F_y = n_{top} - mg = -m\frac{v^2}{r}$$

$$n_{top} = mg - m\frac{v^2}{r} = mg\left(1 - \frac{v^2}{rg}\right)$$

$$n_{top} = mg\left(1 - \frac{3^2}{(10)(9.8)}\right) = 0.908\,mg$$

Ex: A small sphere of mass *m* is attached to the end of a cord of length *R* and set into motion in a vertical circle about a fixed point O as illustrated in figure below. (a) Determine the tangential acceleration of the sphere and the tension in the cord at any instant when the speed of the sphere is *v* and the cord makes an angle *θ* with the vertical. (b) What speed would the ball have as it passes over the top of the circle if the tension in the cord goes to zero instantaneously at this point?

Soln:

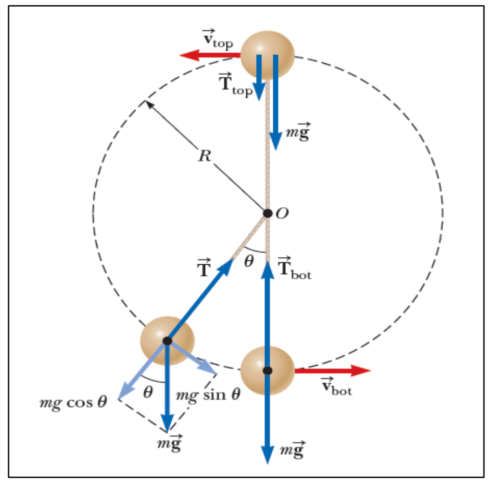
a)

Apply Newton's second law to the sphere in the tangential direction,

$$\sum F_t = mg\sin\theta = ma_t \quad \rightarrow \quad a_t = g\sin\theta$$

Apply Newton's second law to the forces acting on the sphere in the radial direction,

$$\sum F_r = T - mg\cos\theta = \frac{mv^2}{r} \quad \rightarrow \quad T = mg\left(\frac{v^2}{Rg} + \cos\theta\right)$$



The tension at the top and bottom of the circular path:

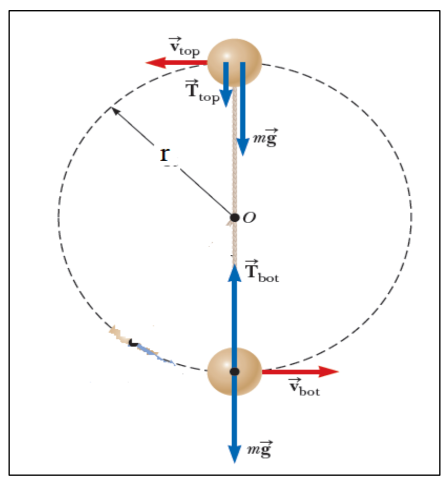
$$T_{top} = mg\left(\frac{v_{top}^2}{Rg} - 1\right)$$

$$T_{bot} = mg\left(\frac{v_{bot}^2}{Rg} + 1\right)$$

b)

$$T_{top} = mg\left(\frac{v_{top}^2}{Rg} - 1\right) = 0 \quad \rightarrow \quad v_{top} = \sqrt{Rg}$$

Ex: A 0.15 kg ball on the end of a 1.1 m long cord (negligible mass) is swung in a vertical circle. (*a*) Determine the minimum speed the ball must have at the top of its arc so that the ball continues moving in a circle. (*b*) Calculate the tension in the cord at the bottom of the arc, assuming the ball is moving at twice the speed of part (*a*).



Soln:

(a) Apply Newton's second law at the top point, for the vertical direction, choosing downward as negative

$$\sum F_y = m\frac{v^2}{r} \rightarrow -T_{top} - mg = -m\frac{v^2}{r}$$

The minimum speed will occur if T_{top} =0

$$mg = m\frac{v^2}{r} \quad \rightarrow \quad v_{min} = \sqrt{gr} = \sqrt{9.8 \times 1.1} = 3.283\,\text{m/s}$$

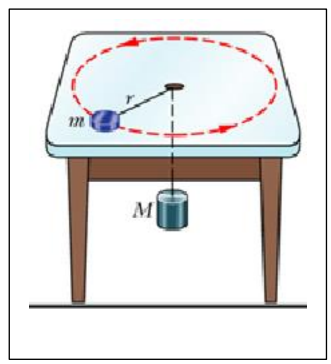
(b) Apply Newton's second law at the bottom, for the vertical direction, choosing upward as positive

$$\sum F_y = m\frac{v^2}{r} \rightarrow T_{bot} - mg = m\frac{v^2}{r}$$

$$T_{bot} = m\left(g + \frac{v^2}{r}\right) = (0.15)\left[(9.8) + \frac{(6.566)^2}{1.10}\right] = 7.35\text{N}$$

Ex: A mass, m, on a frictionless table is attached to a hanging mass, M, by a cord through a hole in the table. Find the speed with which m must move in order for M to stay at rest.

Soln:



Apply Newton's second law to the mass m in the radial direction

$$\sum F_r = T = \frac{mv^2}{r} \qquad \ldots\ldots(1)$$

Apply Newton's second law to the mass M in the vertical direction,

$$\sum F_y = T - Mg = 0 \rightarrow T = Mg \qquad \ldots\ldots.(2)$$

Equate equation (1) and (2): $\rightarrow$ $v = \sqrt{\dfrac{Mgr}{m}}$

Ex: A 40 kg child swings in a swing supported by two chains, each 3 m long. The tension in each chain at the lowest point is 350 N. Find (a) the child's speed at the lowest point and (b) the force exerted by the seat on the child at the lowest point. (Ignore the mass of the seat.)

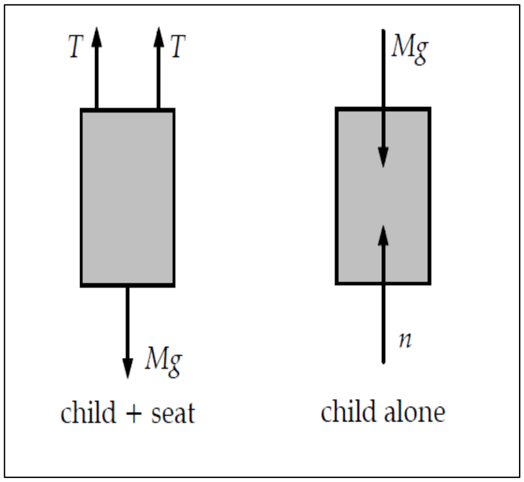
Soln:
Apply Newton's second law on the child and the seat in the vertical direction

$$\sum F_y = 2T - Mg = \frac{Mv^2}{r} \rightarrow v^2 = (2T - Mg)\left(\frac{r}{M}\right) \rightarrow v = \sqrt{(700 - 40 \times 9.8)\left(\frac{3}{40}\right)} = 4.81 \text{ m/s}$$

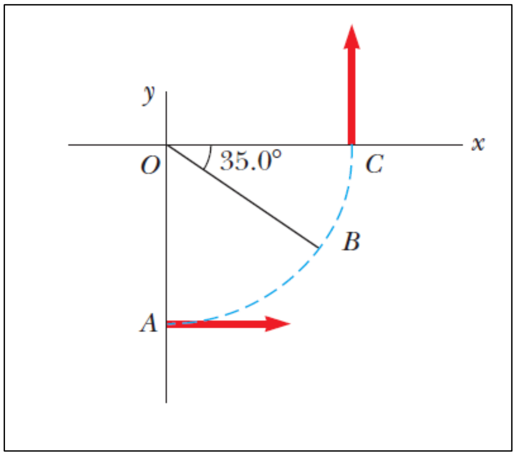
Apply Newton's second law on the child only in the vertical direction

$$\sum F_y = n - Mg = \frac{Mv^2}{r}$$

$$n = \frac{Mv^2}{r} + Mg = 40\left(9.8 + \frac{23.1}{3}\right) = 700\ \text{N}$$



Ex: A car initially traveling eastward turns north by traveling in a circular path at uniform speed as shown in figure below. The length of the arc ABC is 235 m, and the car completes the turn in 36 s. (a) Determine the car's average speed. (b) What is the acceleration when the car is at B located at an angle of 35°? Express your answer in terms of the unit vectors $\hat{i}$ and $\hat{j}$, and (c) its average acceleration during the 36 s interval.



Soln:

a)

$$v = \frac{235}{36} = 6.5\ \text{m/s}$$

b)

$$\frac{1}{4}(2\pi r) = 235\ \text{m} \quad \rightarrow \quad r = 150\ \text{m}$$

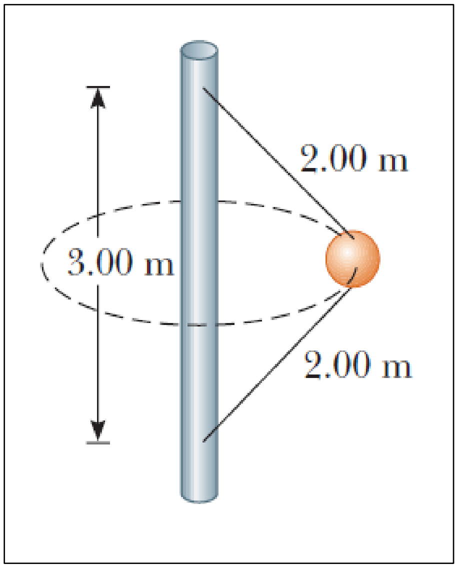
$$a_r = \frac{v^2}{r} = \frac{(6.5)^2}{150} = 0.28\ \text{m/s}^2$$

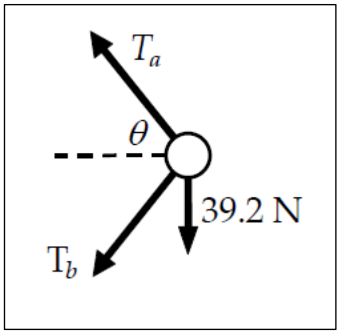
$$\vec{a} = \left(-0.28\cos(35)\hat{i} + 0.28\sin(35)\hat{j}\right)\text{m/s}^2 = \left(-0.23\hat{i} + 0.16\hat{j}\right)\text{m/s}^2$$

c)

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{t} = \frac{6.5\hat{j} - 6.5\hat{i}}{36} = \left(-0.18\hat{i} + 0.18\hat{j}\right)\text{m/s}^2$$

Ex: A 4 kg object is attached to a vertical rod by two strings as shown in figure below. The object rotates in a horizontal circle at constant speed 6 m/s. Find the tension in (a) the upper string and (b) the lower string.





Soln:

$$\sin\theta = \frac{1.5}{2} = 0.75 \rightarrow \quad \theta = 48.6^{o}$$

$$r = 2\times\cos 48.6 = 1.32\ \mathrm{m}$$

Apply Newton's second law in the radial direction

$$\sum F_r = \frac{mv^2}{r} \rightarrow \quad T_a\cos(48.6) + T_b\cos(48.6) = \left(\frac{4\times6^2}{1.32}\right)$$

$$T_a + T_b = \frac{39.2}{\cos(48.6)} = 165\ \mathrm{N} \qquad \text{...........(1)}$$

Apply Newton's second law in the vertical direction

$$\sum F_y = 0 \rightarrow \quad T_a\sin(48.6) - T_b\sin(48.6) - mg = 0$$

$$T_a - T_b = \frac{mg}{\sin(48.6)} = \frac{4\times9.8}{\sin(48.6)} = 52.3\ \mathrm{N} \qquad \text{........(2)}$$

Combine equation (1) and (2):

$$T_a + T_b + T_a - T_b = 165 + 52.3$$

$$T_a = \frac{217}{2} = 108\ \mathrm{N}$$

$$T_b = 165 - T_a = 165 - 108 = 57\ \mathrm{N}$$