4-8- Forces of Friction

When a force $\vec{F}$ tends to slide a body along a surface, a frictional force from the surface acts on the body. The frictional force is parallel to the surface and directed so as to oppose the sliding. It is due to bonding between the body and the surface. If the body does not slide, the frictional force is a ***static frictional force*** $\vec{f}_s$, The magnitude of $\vec{f}_s$ has a maximum value $f_{s,max}$, given by:

$$f_{s,max} = \mu_s n, \qquad \text{.....................(4-8-1)}$$

where μ_s, is the coefficient of static friction and n is the magnitude of the normal force.

If there is sliding, the frictional force is a ***kinetic frictional force*** $\vec{f}_k$. The magnitude of $\vec{f}_k$ is given by:

$$f_k = \mu_k n \qquad \text{.....................(4-8-2)}$$

Where μ_k, is the coefficient of kinetic friction.

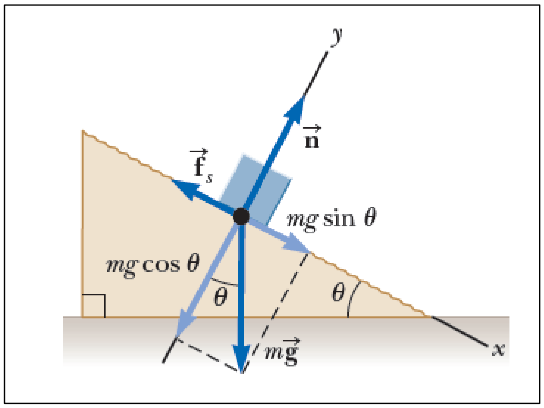
Ex: Suppose a block is placed on a rough surface inclined relative to the horizontal, as shown in figure. The incline angle is increased until the block starts to move. Show that by measuring the critical angle θ_c at which this slipping just occurs, we can obtain μ_s.

Soln:

$$\sum F_x = 0 \rightarrow mg\sin\theta - f_s = 0 \rightarrow f_s = mg\sin\theta \qquad ....(1)$$

$$\sum F_y = 0 \rightarrow n - mg\cos\theta = 0 \rightarrow mg = \frac{n}{\cos\theta} \qquad .....(2)$$

Substitute equation (2) into equation (1):



$$f_s = \frac{n}{\cos\theta}\sin\theta = n\tan\theta$$

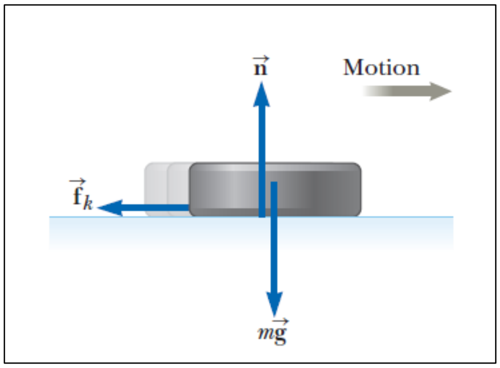
$$\mu_s = \tan\theta_c$$

Ex: A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.

Soln:

$$\sum F_y = 0 \quad \rightarrow \quad n - mg = 0 \quad \rightarrow \quad n = mg \qquad \ldots.(1)$$

$$\sum F_x = ma_x \quad \rightarrow \quad -f_k = ma_x \quad \rightarrow \quad -n\mu_k = ma_x \qquad \ldots.(2)$$



Substitute equation (1) into equation (2):

$$-mg\mu_k = ma_x \quad \rightarrow \quad a_x = -\mu_k g$$

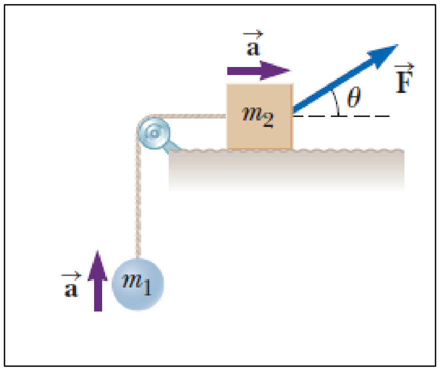
$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \quad , \qquad v_{xf} = 0, \quad v_{xi} = 20 m/s, \quad x_i = 0, \quad x_f = 115 m \ ,$$

$$0 = v_{xi}^2 - 2\mu_k g x_f \quad \rightarrow \quad \mu_k = \frac{v_{xi}^2}{2gx_f} \quad \rightarrow \quad \mu_k = \frac{(20)^2}{2(9.8)(115)} = 0.177$$

Ex: A block of mass *m₂* on a rough, horizontal surface is connected to a ball of mass *m₁* by a lightweight cord over a lightweight, frictionless pulley as shown in figure below. A force of magnitude *F* at an angle θ with the horizontal is applied to the block, and the block slides to the right. The coefficient of kinetic friction between the block and surface is μ_k. Determine the magnitude of the acceleration of the two objects.

Soln:

Apply Newton's second law to the block



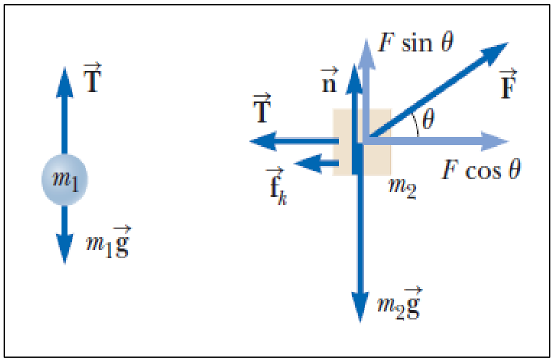
$$\sum F_x = ma_x \quad \rightarrow \quad F\cos\theta - f_k - T = m_2 a \qquad \ldots.(1)$$

$$\sum F_y = ma_y \quad \rightarrow \quad n + F\sin\theta - m_2 g = 0 \quad \rightarrow \quad n = m_2 g - F\sin\theta \qquad \ldots.(2)$$

$$f_k = \mu_k \left(m_2 g - F\sin\theta\right) \qquad \ldots.(3)$$

Apply Newton's second law to the ball

$$\sum F_y = ma_y \quad \rightarrow \quad T - m_1 g = m_1 a \quad \rightarrow \quad T = m_1(g + a) \qquad \ldots.(4)$$



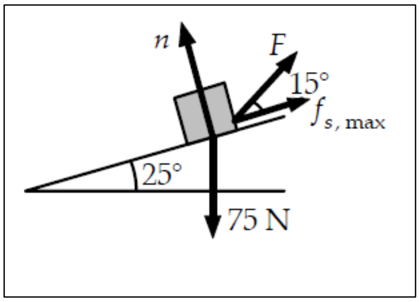
Substitute equation (4) and (3) into (1):

$$F\cos\theta - \mu_k \left(m_2 g - F\sin\theta\right) - m_1(g + a) = m_2 a$$

$$a = \frac{F\left(\cos\theta + \mu_k \sin\theta\right) - \left(m_1 + \mu_k m_2\right)g}{m_1 + m_2}$$

Ex: A block weighing 75 N rests on a plane inclined at 25° to the horizontal. A force F is applied to the object at 15° to the horizontal, pushing it upward on the plane. The coefficients of static and kinetic friction between the block and the plane are, respectively, 0.363 and 0.156. (a) What is the minimum value of F that will prevent the block from slipping down the plane? (b) What is the minimum value of F that will start the block moving up the plane? (c) What value of F will move the block up the plane with constant velocity?

Soln:



(a)

a)

$$\sum F_x = 0$$

$$F\cos(15) + f_s - 75\sin(25) = 0$$

$$f_{s,max} = 75\sin(25) - F\cos(15) \quad \text{........(1)}$$

$$\sum F_y = 0$$

$$n + F\sin(15) - 75\cos(25) = 0$$

$$n = 75\cos(25) - F\sin(15)$$

$$f_{s,max} = 75\mu_s\cos(25) - F\mu_s\sin(15) \quad \text{......(2)}$$

Equate equation (1) and (2):

$$75\sin(25) - F\cos(15) = 75\mu_s\cos(25) - F\mu_s\sin(15)$$

$$F[\cos(15) - \mu_s\sin(15)] = [75\sin(25) - 75\mu_s\cos(25)] \rightarrow F = \frac{[75\sin(25) - 75\mu_s\cos(25)]}{[\cos(15) - \mu_s\sin(15)]} = 8\text{N}$$

b)

$$\sum F_x = 0$$

$$F\cos(15) - f_s - 75\sin(25) = 0$$

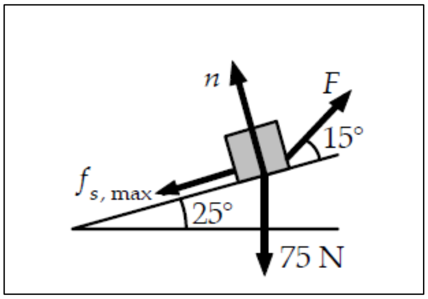
$$f_{s,max} = F\cos(15) - 75\sin(25) \quad \text{........(3)}$$

$$\sum F_y = 0$$

$$n + F\sin(15) - 75\cos(25) = 0$$

$$n = 75\cos(25) - F\sin(15)$$

$$f_{s,max} = 75\mu_s\cos(25) - F\mu_s\sin(15) \quad \text{......(4)}$$



(b)

Equate equation (3) and (4):

$$F\cos(15) - 75\sin(25) = 75\mu_s\cos(25) - F\mu_s\sin(15)$$

$$F[\cos(15) + \mu_s\sin(15)] = [75\sin(25) + 75\mu_s\cos(25)] \rightarrow F = \frac{[75\sin(25) + 75\mu_s\cos(25)]}{[\cos(15) + \mu_s\sin(15)]} = 53.2\text{N}$$

c)

$$\sum F_x = 0$$

$$F\cos(15) - f_k - 75\sin(25) = 0$$

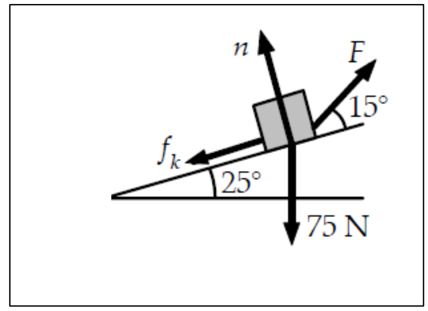
$$f_k = F\cos(15) - 75\sin(25) \qquad \text{........(5)}$$

$$\sum F_y = 0$$

$$n + F\sin(15) - 75\cos(25) = 0$$

$$n = 75\cos(25) - F\sin(15)$$

$$f_k = 75\mu_k\cos(25) - F\mu_k\sin(15) \qquad \text{......(6)}$$



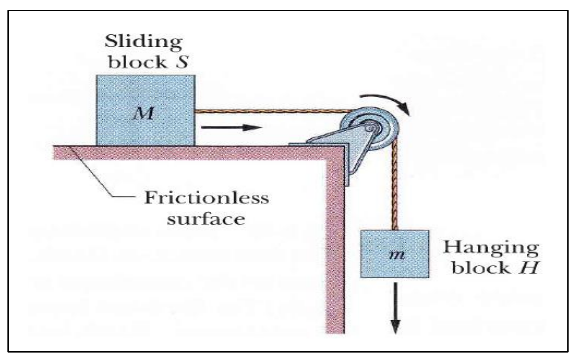
(c)

Equate equation (5) and (6):

$$F\cos(15) - 75\sin(25) = 75\mu_s\cos(25) - F\mu_s\sin(15)$$

$$F[\cos(15) + \mu_k\sin(15)] = [75\sin(25) + 75\mu_k\cos(25)] \quad \rightarrow \quad F = \frac{[75\sin(25) + 75\mu_k\cos(25)]}{[\cos(15) + \mu_k\sin(15)]} = 42\text{N}$$

Ex: Figure below shows a block S (the sliding block) with mass M. The block is free to move along a horizontal frictionless surface and connected, by a cord that wraps over a frictionless pulley, to a second block H (the hanging block), with mass m. The cord and pulley have negligible masses compared to the blocks. The hanging block H falls as the sliding block S accelerates to the right. Find (a) the acceleration of the two blocks, (b) the tension in the cord.



Soln:

Apply Newton's second law to the block S:

$$\sum F_x = Ma_x \rightarrow T = Ma$$

Apply Newton's second law to the block H:

$$\sum F_y = ma_y \rightarrow T - mg = -ma$$

$$a = \frac{mg}{M+m}$$

$$T = \frac{mM}{M+m}g$$