Chapter 4: The Laws of Motion

4-1- The Concept of Force

A force \vec{F} is defined as an interaction between two bodies or between a body and its environment. A force is a vector quantity, and the SI unit of the magnitude of the force is the newton (N). The net force $\sum \vec{F}$ acting on an object is defined as the vector sum of all forces acting on the object.

We sometimes refer to the net force as the total force or the resultant force. If the net force exerted on an object is zero, the acceleration of the object is zero and the object either remains at rest or continues to move with constant velocity. When an object is either at rest or moving with constant velocity (in a straight line with constant speed), we say that the object is in equilibrium.

There are two classes of forces:

1) Contact forces, when a force involves direct contact between two objects.



2) *Field forces*, they do not involve physical contact between two objects. These forces act through empty space.



4-2- Newton's first law

An object at rest stays at rest and an object in motion stays in motion at a constant speed and direction unless acted upon by an external force.

$$\sum \vec{F} = 0$$
 (Object in equilibrium)(4-2-1)

For this to be true, each component of the net force must be zero, so

$$\begin{split} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0 \end{split} \tag{Object in equilibrium} \qquad(4-2-2) \end{split}$$

Newton's first law of motion sometimes called the law of inertia.

The tendency of an object to resist any attempt to change its state (either at rest or in motion) is called inertia.

4-3- Frame of reference:

- A reference frame is a space in which we are making observations and measuring physical quantities.
- A reference frame is either at rest or moves with constant velocity is called inertial frame of reference. A reference frame that is accelerating in either linear fashion or rotating around some axis is called non inertial reference frame.
- Any reference frame that moves with constant velocity relative to an inertial frame is itself an inertial frame.
- > Newton's first law is only valid in inertial frame of reference.

4-4- Mass

- > The mass is the amount of matter in an object and is an intrinsic characteristic of the body.
- Mass is that property of an object that specifies how much resistance an object exhibits to changes in its velocity.
- The greater the mass of an object, the less that object accelerates under the action of a given applied force.
- $\blacktriangleright Mass is a scalar quantity.$

4-5- Newton's second law

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

Equation (4-5-1) is equivalent to three component equations:

4-6- The gravitational force and weight

- > The attractive force exerted by the Earth on an object is called the gravitational force and is directed toward the center of the Earth, and is given by: $\vec{F}_g = m\vec{g}$.
- > The magnitude of the gravitational force is called the weight of an object $F_g = mg$

4-7- Newton's third law

If two objects interact, the force \vec{F}_{12} exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force \vec{F}_{21} exerted by object 2 on object 1:

$$\vec{F}_{12} = -\vec{F}_{21}$$

Ex: A worker applies a constant horizontal force with magnitude 20 N to a box with mass 40 kg resting on a level floor with negligible friction. What is the acceleration of the box?

Soln: $\sum F_x = ma_x$

$$a_x = \frac{\sum F_x}{m} = \frac{20 \text{ N}}{40 \text{ kg}} = 0.5 \text{ m/s}^2$$



Ex: A hockey puck having a mass of 0.30 kg slides on horizontal, frictionless surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck. The force \vec{F}_1 has a magnitude of 5.0 N, and is directed at $\theta = 20^\circ$ below the x-axis and the force \vec{F}_2 has a magnitude of 8.0 N and is directed at $\varphi = 60^\circ$ above the x-axis. Determine both the magnitude and the direction of the puck's acceleration.

Soln:

$$\sum F_x = ma_x \quad \Rightarrow \quad a_x = \frac{\sum F_x}{m} \quad \Rightarrow \quad a_x = \frac{F_{1x} + F_{2x}}{m}$$
$$a_x = \frac{F_1 \cos(-20^\circ) + F_2 \cos(60^\circ)}{m}$$

$$a_{x} = \frac{(5 \text{ N})\cos(-20^{\circ}) + (8 \text{ N})\cos(60^{\circ})}{0.3 \text{ kg}} = \frac{8.7 \text{ N}}{0.3 \text{ kg}} = 29 \text{ m/s}^{2}$$

$$\sum F_{y} = ma_{y} \implies a_{y} = \frac{\sum F_{y}}{m} \implies a_{y} = \frac{F_{1y} + F_{2y}}{m}$$
$$a_{y} = \frac{F_{1} \sin(-20^{\circ}) + F_{2} \sin(60^{\circ})}{m}$$



$$a_{y} = \frac{(5 \text{ N})\sin(-20^{\circ}) + (8 \text{ N})\sin(60^{\circ})}{0.3 \text{ kg}} = \frac{5.2 \text{ N}}{0.3 \text{ kg}} = 17 \text{ m/s}^{2}$$
$$a = \sqrt{a_{x}^{2} + a_{y}^{2}} = \sqrt{(29)^{2} + (17)^{2}} = 34 \text{ m/s}^{2}$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^{\circ}$$

Ex: A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support, as in the figure below. The upper cables make angles of 37.0° and 53.0° with the horizontal. These upper cables are not as strong as the vertical cable, and will break if the tension in them exceeds 100 N. Will the traffic light remain hanging in this situation, or will one of the cables break?



Soln:

Apply equilibrium condition for the traffic light in the y-direction

$$\sum F_{y} = 0 \quad \Rightarrow \quad T_{3} - F_{g} = 0 \quad \Rightarrow \quad T_{3} = F_{g} = 122 \text{ N}$$

Apply the equilibrium condition to the knot in the x-direction:

Apply the equilibrium condition to the knot in the y-direction:

Substitute equation (1) into (2)

$$T_1 \sin 37^\circ + (1.33 T_1) \sin 53^\circ - 122 N = 0 \rightarrow T_1 = 73.4 N$$

$$T_2 = 1.33 T_1 \rightarrow T_2 = (1.33)(73.4) = 97.4 N$$

Both T_1 and T_2 are less than 100 N, so the cables will not break.

Ex: A car of mass m is on an icy driveway inclined at an angle θ . (a) Find the acceleration of the car, assuming that the driveway is frictionless. (b) Suppose the car is released from rest at the top of the incline, and the distance from the car's front bumper to the bottom of the incline is d. How long does it take the front bumper to reach the bottom, and what is the car's speed as it arrives there?

Soln:

a) Apply Newton's second law in the x-direction:

 $\sum F_{x} = ma_{x} \rightarrow mg \sin \theta = ma_{x} \rightarrow a_{x} = g \sin \theta$ $\sum F_{y} = ma_{y} \rightarrow n - mg \cos \theta = 0 \rightarrow n = mg \cos \theta$ b) $x_{f} - x_{i} = v_{xi}t + \frac{1}{2}a_{x}t^{2}$ $v_{xi} = 0, \quad x_{i} = 0 \text{ and } x_{f} = d$ $d = \frac{1}{2}a_{x}t^{2} \rightarrow t = \sqrt{\frac{2d}{a_{x}}} \rightarrow t = \sqrt{\frac{2d}{g \sin \theta}}$ $v_{xf} = v_{xi} + a_{x}t \rightarrow v_{xf} = 0 + (g \sin \theta)\sqrt{\frac{2d}{g \sin \theta}} = \sqrt{2gd \sin \theta}$



Ex: Two blocks of masses m_1 and m_2 , with $m_1 > m_2$, are placed in contact with each other on a frictionless, horizontal surface. A constant horizontal force \vec{F} is applied to m_1 as shown below. (a) Find the magnitude of the acceleration of the system. (b) Determine the magnitude of the contact force between the two blocks.

Soln:

a) Apply Newton's Second law to the combination

$$\sum F_x = ma_x \rightarrow F = (m_1 + m_2)a_x \rightarrow a_x = \frac{F}{m_1 + m_2}$$

b) Apply Newton's second law to the m2



$$\sum F_{x} = ma_{x} \rightarrow P_{12} = m_{2}a_{x} \rightarrow P_{12} = \left(\frac{m_{2}}{m_{1} + m_{2}}\right)F$$

Apply Newton's second law to the m1

$$\sum F_{x} = ma_{x} \rightarrow F - P_{21} = m_{1}a_{x} \rightarrow P_{21} = F - m_{1}a_{x} \rightarrow P_{21} = F - \left(\frac{m_{1}}{m_{1} + m_{2}}\right)F \rightarrow P_{21} = \left(\frac{m_{2}}{m_{1} + m_{2}}\right)F$$

 $P_{21} = P_{12}$

Ex: A person weighs a fish of mass m on a spring scale attached to the ceiling of an elevator, as illustrated in figure below. a) Show that if the elevator accelerates either upward or downward, the spring scale gives a reading that is different from the weight of the fish. b) Evaluate the scale readings for a 40 N fish if the elevator moves with an acceleration $a_y=\pm 2.00 \text{ m/s}^2$.



Soln:

a) If the elevator moves with an acceleration \vec{a} relative to an observer standing outside the elevator in an inertial frame.

Newton's second law applied to the fish gives the net force on the fish:

We conclude from equation (1) that:

- The scale reading T is greater than the fish's weight mg if \vec{a} is upward, so that a_y is positive.
- The reading is less than mg if \overline{a} is downward, so that a_y is negative.

b)

If a_y=2.00 m/s²
$$\rightarrow$$
 T = (40 N) $\left(\frac{2 \text{ m/s}^2}{9.8 \text{ m/s}^2} + 1\right)$ = 48.2 N

If a_y=-2.00 m/s²
$$\rightarrow$$
 T = $(40 \text{ N}) \left(\frac{-2 \text{ m/s}^2}{9.8 \text{ m/s}^2} + 1 \right) = 31.8 \text{ N}$

Ex: Determine the magnitude of the acceleration of the two objects and the tension in the lightweight cord, were $m_2 > m_1$, as shown in the figure below.

Soln:

If we define the upward direction as positive for object 1, we must define the downward direction as positive for object 2.

Apply Newton's second law to object 1

$$\sum F_y = ma_y \rightarrow T - m_1g = m_1a_y$$
(1)

Apply Newton's second law to object 2

$$\sum F_y = ma_y \rightarrow m_2g - T = m_2a_y$$
(2)

Add equation (1) to (2), we get

$$(m_2 - m)_1 g = (m_1 + m_2) a_y$$

Solve for the acceleration:

$$\mathbf{a}_{\mathbf{y}} = \left(\frac{\mathbf{m}_2 - \mathbf{m}_1}{\mathbf{m}_1 + \mathbf{m}_2}\right) \mathbf{g}$$

When the last equation is substituted into (1), we obtain

$$\mathbf{T} - \mathbf{m}_{1}\mathbf{g} = \mathbf{m}_{1}\mathbf{a}_{y} \quad \Rightarrow \quad \mathbf{T} = \mathbf{m}_{1}\left(\mathbf{a}_{y} + \mathbf{g}\right) \quad \Rightarrow \quad \mathbf{T} = \mathbf{m}_{1}\left(\left(\frac{\mathbf{m}_{2} - \mathbf{m}_{1}}{\mathbf{m}_{1} + \mathbf{m}_{2}}\right)\mathbf{g} + \mathbf{g}\right) \quad \Rightarrow \quad \mathbf{T} = \mathbf{g}\left(\frac{2\mathbf{m}_{1}\mathbf{m}_{2}}{\mathbf{m}_{1} + \mathbf{m}_{2}}\right)\mathbf{g} + \mathbf{g}$$



Ex: A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass, as in figure below. The block lies on a frictionless incline of angle θ . Find the magnitude of the acceleration of the two objects and the tension in the cord.



Soln:

Applying Newton's second law in component form to the ball, choosing the upward direction as positive,

$$\sum F_{y} = ma_{y} \rightarrow T - m_{1}g = m_{1}a_{y}$$
$$T - m_{1}g = m_{1}a \rightarrow T = m_{1}(g + a) \qquad \dots \dots \dots (2)$$

Apply Newton's second law in component form to the block. It is convenient to choose the positive x'-axis along the incline, we choose the positive direction to be down the incline.

$$\sum F_{x'} = ma_{x'} \rightarrow m_2 g \sin \theta - T = m_2 a_{x'} \rightarrow m_2 g \sin \theta - T = m_2 a \dots (3)$$
$$\sum F_{x'} = ma_{x'} \rightarrow n - m_2 g \cos \theta = 0 \dots (4)$$

Substitute equation (2) into (3) and solve for a:

$$m_2g\sin\theta - m_1(g+a) = m_2a \rightarrow m_2g\sin\theta - m_1g = (m_1 + m_2)a \rightarrow a = \frac{m_2g\sin\theta - m_1g}{m_1 + m_2}$$

Substitute into equation (2), we get:

$$T = m_1(g+a) \rightarrow T = m_1\left(g + \frac{m_2g\sin\theta - m_1g}{m_1 + m_2}\right) \rightarrow T = \frac{m_1m_2g(\sin\theta + 1)}{m_1 + m_2}$$

Ex: In the overhead view of figure below, a 2.0 kg cookie tin is accelerated at 3.0 m/s² in the direction shown by \vec{a} , over a frictionless horizontal surface. The acceleration is caused by three horizontal forces, only two of which are shown: \vec{F}_1 of magnitude 10 N and \vec{F}_2 of magnitude 20 N. What is the third force \vec{F}_3 in unit vector notation and in magnitude - angle notation?

Soln:

Apply Newton's second law:
$$\sum \vec{F} = m\vec{a}$$

 $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a}$
 $\vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2$
The x-component:
 $F_{3,x} = ma_x - F_{1,x} - F_{2,x}$
 $= m(a \cos 50^\circ) - F_1 \cos(-150^\circ) - F_2 \cos 90^\circ$.
 $F_{3,x} = (2.0 \text{ kg})(3.0 \text{ m/s}^2) \cos 50^\circ - (10 \text{ N}) \cos(-150^\circ)$
 $- (20 \text{ N}) \cos 90^\circ$
 $= 12.5 \text{ N}.$



The y-component:

$$F_{3,y} = ma_y - F_{1,y} - F_{2,y}$$

= $m(a \sin 50^\circ) - F_1 \sin(-150^\circ) - F_2 \sin 90^\circ$
= $(2.0 \text{ kg})(3.0 \text{ m/s}^2) \sin 50^\circ - (10 \text{ N}) \sin(-150^\circ)$
 $- (20 \text{ N}) \sin 90^\circ$
= $-10.4 \text{ N}.$
 $\vec{F}_3 = F_{3,x}\hat{i} + F_{3,y}\hat{j} = (12.5 \text{ N})\hat{i} - (10.4 \text{ N})\hat{j}$

$$F_3 = \sqrt{F_{3,x}^2 + F_{3,y}^2} = 16 \text{ N}$$

$$\theta = \tan^{-1} \frac{F_{3,y}}{F_{3,x}} = -40^{\circ}.$$



Ex: Besides the gravitational force, a 2.80 kg object is subjected to one other constant force. The object starts from rest and in 1.20 s experiences a displacement of $(4.2\hat{i}-3.3\hat{j})$ m. Determine the other force.

Soln:

$$\begin{aligned} \vec{r}_{f} &- \vec{r}_{i} = \vec{v}_{i}t + \frac{1}{2}\vec{a}t^{2} \\ 4.2\hat{i} - 3.3\hat{j} &= 0 + \frac{1}{2}\vec{a}(1.2)^{2} \quad \Rightarrow \quad \vec{a} = (5.83\hat{i} - 4.58\hat{j})m/s^{2} \\ \sum \vec{F} &= m\vec{a} \quad \Rightarrow \quad \vec{F}_{g} + \vec{F} = m\vec{a} \quad \Rightarrow \quad \vec{F} = m\vec{a} - \vec{F}_{g} = (2.8)(5.83\hat{i} - 4.58\hat{j}) - (2.8)(-9.8\hat{j}) = (16.3\hat{i} + 14.6\hat{j}) N \end{aligned}$$

Ex: A bag of cement weighing 325 N hangs in equilibrium from three wires as suggested in figure below. Two of the wires make angles $\theta_1 = 60^\circ$ and $\theta_2 = 25^\circ$ with the horizontal. Assuming the system is in equilibrium, find the tensions T_1 , T_2 , and T_3 in the wires.

Soln:

Apply equilibrium condition for the bag in the y-direction

 $\sum F_y = 0 \rightarrow T_3 - F_g = 0 \rightarrow T_3 = F_g = 325 \text{ N}$

Apply the equilibrium condition to the knot in the x-direction:

$$\sum F_{x} = 0 \quad \Rightarrow \quad -T_{1} \cos \theta_{1} + T_{2} \cos \theta_{2} = 0 \quad \Rightarrow \quad T_{2} = T_{1} \left(\frac{\cos \theta_{1}}{\cos \theta_{2}} \right)$$

Apply the equilibrium condition to the knot in the y-direction:

$$\sum F_{y} = 0 \quad \Rightarrow \quad T_{1} \sin \theta_{1} + T_{2} \sin \theta_{2} - F_{g} = 0$$

Eliminate T₂ and solve for T₁

$$T_1 = \frac{F_g \cos \theta_2}{\sin \theta_1 \cos \theta_2 + \cos \theta_1 \sin \theta_2} = \frac{(325) \cos 25^\circ}{\sin (85^\circ)} = 296 \text{ N}$$

$$T_2 = (296) \left(\frac{\cos 60^\circ}{\cos 25^\circ} \right) = 163 \text{ N}$$



