2-6- One dimensional motion with constant acceleration

If the acceleration of a particle varies in time, its motion can be complex and difficult to analyze. However, a very common and simple type of one-dimensional motion is that in which the acceleration is constant.

If the acceleration is constant, then the average acceleration is equal to instantaneous acceleration, i.e

 $\overline{a}_x = a_x$

If we take $t_i=0$ and t_f to be any later time t, we find that

A velocity-time graph for this constant-acceleration motion is shown in the following Figure 2.1(a). When the acceleration is constant, the graph of acceleration versus time Figure 2.1(b) is a straight line having a slope of zero.



Because velocity at constant acceleration varies linearly in time, we can express the average velocity in any time interval as

$$\overline{v}_{x} = \frac{v_{xf} + v_{xi}}{2}$$
 (for constant a_x) ------ (2)

Recalling that $\overline{v}_x = \frac{x_f - x_i}{t_f - t_i} = \frac{x_f - x_i}{t}$

Equating the above equation with equation (2), we get:

$$x_{f} - x_{i} = \frac{v_{xf} + v_{xi}}{2}t \qquad (\text{for constant } a_{x}) \qquad \dots \qquad (3)$$

We can obtain another useful expression for displacement at constant acceleration by substituting equation (1) into equation (3): $x_f - x_i = \frac{1}{2}(2v_{xi} + a_x t)t$

$$x_{f} - x_{i} = v_{xi}t + \frac{1}{2}a_{x}t^{2}$$
 (for constant a_{x}) ------ (4)

Finally, we can obtain an expression for the final velocity that does not contain a time interval by substituting the value of "t" from equation (1) into equation (3):

$$\begin{aligned} \mathbf{x}_{\mathrm{f}} - \mathbf{x}_{\mathrm{i}} &= \left(\frac{\mathbf{v}_{\mathrm{xf}} + \mathbf{v}_{\mathrm{xi}}}{2}\right) \left(\frac{\mathbf{v}_{\mathrm{xf}} - \mathbf{v}_{\mathrm{xi}}}{\mathbf{a}_{\mathrm{x}}}\right) \\ \mathbf{x}_{\mathrm{f}} - \mathbf{x}_{\mathrm{i}} &= \left(\frac{\mathbf{v}_{\mathrm{xf}}^{2} - \mathbf{v}_{\mathrm{xi}}^{2}}{2\mathbf{a}_{\mathrm{x}}}\right) \\ \mathbf{v}_{\mathrm{xf}}^{2} &= \mathbf{v}_{\mathrm{xi}}^{2} + 2\mathbf{a}_{\mathrm{x}}\Delta\mathbf{x} \end{aligned}$$
 (for constant a_x) ----- (5)

Ex: A jet lands on an aircraft carrier at 63 m/s. (a) What is its acceleration if it stops in 2 s?(b) What is the displacement of the plane while it is stopping?

Soln:

(a)
$$a_x = \frac{v_{xf} - v_{xi}}{t} = \frac{0 - 63}{2} = -31.5 \text{ m/s}^2$$

(b)
$$x_{f} - x_{i} = \frac{v_{xf} + v_{xi}}{2} t = \left(\frac{0+63}{2}\right)(2) = 63m$$

Ex: A car traveling at a constant speed of 45.0 m/s passes a trooper on a motorcycle hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch the car, accelerating at a constant rate of 3.0 m/s^2 . How long does it take the trooper to overtake the car?

Soln:

Choose the position of the billboard as the origin $x_B = 0$

For car
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_xt^2 \rightarrow x_{car} = 45m + (45 m/s)t + 0 = 45 + 45t$$

For trooper $x_f = x_i + v_{xi}t + \frac{1}{2}a_xt^2 \rightarrow x_{trooper} = 0 + (0)t + \frac{1}{2}(3)t^2 = 1.5t^2$

The trooper overtaking the car when $x_{trooper} = x_{car}$

 $45 + 45 t = 1.5t^2$

This gives the quadratic equation $1.5t^2 - 45t - 45 = 0$

$$t = \frac{45 + \sqrt{45^2 + 4(1.5)(45)}}{2(1.5)} = 31 \text{ s.}$$

2-7- Freely falling objects

In the absence of air resistance, all objects dropped near the Earth's surface fall toward the Earth with the same constant acceleration under the influence of the Earth's gravity. **A freely falling object** is any object moving freely under the influence of gravity alone, regardless of its initial motion. Objects thrown upward or downward and those released from rest are all falling freely once they are released. Any freely falling object experiences acceleration directed downward, regardless of its initial motion.

It is common to define "up" as the +y direction and to use y as the position variable in the kinematic equations. At the Earth's surface, the value of g is approximately 9.80 m/s². The equations developed above for objects moving with constant acceleration can be applied. The only modification that we need to make in these equations for freely falling objects is to note that the motion is in the vertical direction (the y direction) rather than in the horizontal (x) direction and that the acceleration is downward and has a magnitude of 9.80 m/s². Thus, we always take a_y =-g=-9.8 m/s², where the minus sign means that the acceleration of a freely falling object is downward.

Ex: A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50 m high, and the stone just misses the edge of the roof on its way down. Using $t_A=0$ as the time the stone leaves the thrower's hand at position A, determine **(a)** the time at which the stone reaches its maximum height, **(b)** the maximum height, **(c)** the time at which the stone returns to the height from which it was thrown, **(d)** the velocity of the stone at this instant, and **(e)** the velocity and position of the stone at t = 5 s.

Soln.

a)

$$v_{yf} = v_{y_1} + a_y t \rightarrow t = \frac{v_{yf} - v_{y_1}}{a_y} \rightarrow t_B = \frac{0 - 20}{-9.8} = 2.04 \text{ s}$$

b)
 $y_{max} = y_B = y_A + v_{yA}t + \frac{1}{2}a_yt^2$
 $y_{max} = y_B = 0 + (20 \text{ m/s})(2.04 \text{ s}) - \frac{1}{2}(9.8)(2.04 \text{ s})^2 = 20.4 \text{ m}$
c)
 $y_c - y_A = v_{yA}t + \frac{1}{2}a_yt^2 \rightarrow 0 - 0 = 20t - \frac{1}{2}(9.8)t^2$
 $t(20 - 4.9t) = 0 \rightarrow t = 4.08 \text{ sec}$
d)
 $v_{yf}^2 = v_{yi}^2 + 2a_y\Delta y \rightarrow v_{yC}^2 = v_{yA}^2 + 2a_y(y_C - y_A)$
 $v_{yC}^2 = (20)^2 + 2(9.8) - (0 - 0) = 400 \text{ m}^2/\text{s}^2$
 $v_{yC} = -20 \text{ m/s}$
e)
 $v_{yf} = v_{yi} + a_yt \rightarrow v_{yD} = v_{yA} + a_yt$

$$v_{yD} = 20 + (-9.8)5 = -29 \text{ m/s}$$

$$y_{\rm D} = y_{\rm A} + v_{\rm yA}t + \frac{1}{2}a_{\rm y}t^2 \rightarrow y_{\rm D} = 0 + (20)(5) + \frac{1}{2}(-9.8)(5)^2 = -22.5 \text{ m}$$