Chapter 2: Motion in one dimension

2-1- Position

A particle's **position** x is the location of the particle with respect to a chosen reference point that we can consider to be the origin of a coordinate system.

2-2- Displacement and Distance

The **displacement** Δx of a particle is defined as its change in position. As it moves from an initial position x_i to a final position x_f, we write the displacement of the particle as

$$\Delta x = x_f - x_i$$

From this definition we see that Δx is positive if x_f is greater than x_i and negative if x_f is less than x_i.

Displacement is an example of a vector quantity. We use plus and minus signs to indicate vector direction. Any object always moving to the right undergoes a positive displacement $+\Delta x$, and any object moving to the left undergoes a negative displacement $-\Delta x$.

It is very important to recognize the difference between displacement and distance traveled.

Distance d is the length of a path followed by a particle.

Ex: What is the difference between distance and displacement?

Displacement	Distance
Vector quantity (has direction and magnitude)	Scalar quantity (has magnitude only)
Positive or negative	Always positive
It's magnitude is shortest length between two points	is the length between two points longer than straight line between them

2-3- Average velocity and speed

The **average velocity** $\bar{v}_x$ of a particle is defined as the particle's displacement Δx divided by the time interval Δt during which that displacement occurred:

$$\bar{v}_x = \frac{\Delta x}{\Delta t}$$

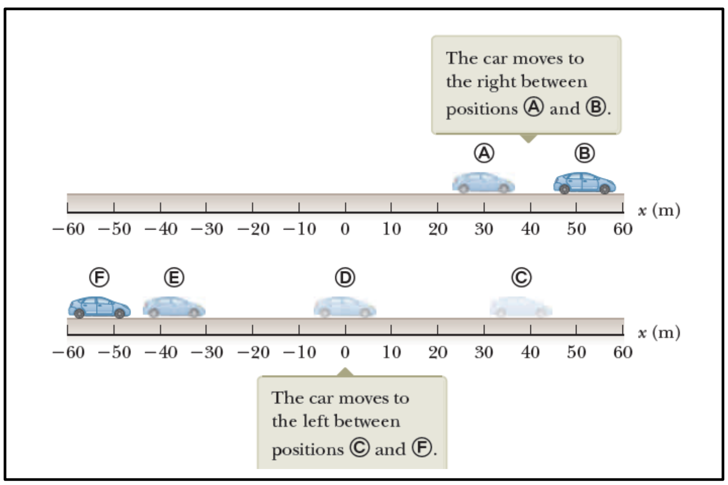
where the subscript “x” indicates motion along the x-axis. The average velocity has dimensions of length divided by time (L/T), or meters per second in SI units.

There is a clear distinction between speed and velocity. The **average speed** $\bar{v}$ of a particle, a scalar quantity, is defined as the total distance traveled $\mathbf{d}$ divided by the total time it takes to travel that distance:

$$\bar{v} = \frac{d}{\Delta t}$$

The SI unit of average speed is the same as the unit of average velocity: meters per second. However, unlike average velocity, average speed has no direction and hence carries no algebraic sign.

Ex: Find the displacement, average velocity, and average speed of the car in the following figure between positions A and F. Note that X_A=30 m at t= 0 sec and that X_F = -53m at t=50 sec



Position	t(s)	x(m)
A	0	30
B	10	52
C	20	38
D	30	0
E	40	-37
F	50	-53

Soln:

$$\Delta x = x_F - x_A = -53\text{m} - 30\text{m} = -83\text{m}$$

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{-53-30}{50-0} = -1.7 \text{ m/s}$$

$$\bar{v} = \frac{d}{\Delta t} = \frac{127}{50} = 2.5\,\text{m/s}$$

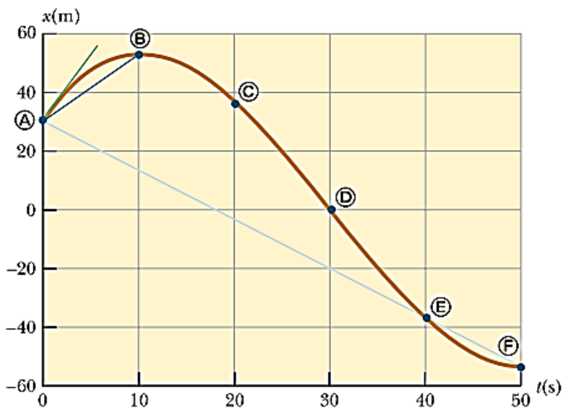
2-4- Instantaneous velocity and speed

Instantaneous velocity v_x equals the limiting value of the ratio Δx/Δt as Δt approaches zero

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The instantaneous velocity can be positive, negative, or zero. In the following figure, when the slope of the position-time graph is positive, such as at any time during the first 10s, v_x is positive. After point B, v_x is negative because the slope is negative. At the peak, the slope and the instantaneous velocity are zero.

The instantaneous speed of a particle is defined as the magnitude of its velocity.

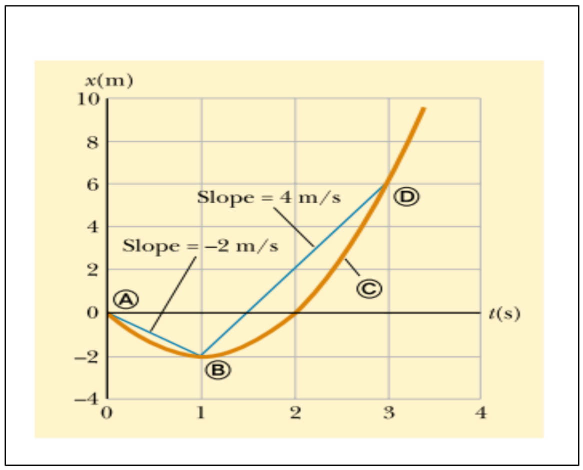


Ex: A particle moves along the x-axis. Its x coordinate varies with time according to the expression $x=-4t+2t^2$ where x is in meters and t is in seconds. The position-time graph for this motion is shown in following figure. Note that the particle moves in the negative x direction for the first second of motion, is at rest at the moment t =1 s, and moves in the positive x direction for t>1 s.

a) Determine the displacement of the particle in the time intervals t=0 to t=1 sec and t=1 s to t=3 s.

b) Calculate the average velocity during these two time intervals.

c) Find the instantaneous velocity of the particle at t=2.5 s.



Soln:

a)

$$\Delta x_{Ⓐ\to Ⓑ} = x_f - x_i = x_{Ⓑ} - x_{Ⓐ}$$
$$= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2] = \boxed{-2\text{ m}}$$
$$\Delta x_{Ⓑ\to Ⓓ} = x_f - x_i = x_{Ⓓ} - x_{Ⓑ}$$
$$= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2] = \boxed{+8\text{ m}}$$

b)

$$\bar{v}_{x\,(Ⓐ\to Ⓑ)} = \frac{\Delta x_{Ⓐ\to Ⓑ}}{\Delta t} = \frac{-2\text{ m}}{1\text{ s}} = \boxed{-2\text{ m/s}}$$

$$\bar{v}_{x\,(Ⓑ\to Ⓓ)} = \frac{\Delta x_{Ⓑ\to Ⓓ}}{\Delta t} = \frac{8\text{ m}}{2\text{ s}} = \boxed{+4\text{ m/s}}$$

c)

$$v_x = \frac{dx}{dt} = -4 + 4t$$
$$v_x(t = 2.5) = 6\text{m/s}$$

2-5- Acceleration

The **average acceleration** $\bar{a}_x$ of the particle is defined as the change in velocity Δv_x divided by the time interval Δt during which that change occurred:

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{x_f} - v_{x_i}}{t_f - t_i}$$

As with velocity, we can use positive and negative signs to indicate the direction of the acceleration. Acceleration has dimensions of length divided by time squared, or L/T^2. The SI unit of acceleration is meters per second squared (m/sec^2).

The **instantaneous acceleration** equals the derivative of the velocity with respect to time

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

If a_x is positive, then the acceleration is in the positive x direction; if a_x is negative, then the acceleration is in the negative x direction. The acceleration can also be written

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Ex: The velocity of a particle moving along the x-axis varies in time according to the expression v_x=(40-5t^2) m/s, where t is in seconds. (a) Find the average acceleration in the time interval t=0 to t= 2.0 s. (b) Determine the acceleration at t = 2.0 s.

Soln:

a)

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

v_{xA}=40-5(0)^2=40 m/s

v_{xB}=40-5(2)^2=20 m/s

$$\bar{a}_x = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{(20-40)\,m/s}{(2-0)s} = -10 \text{ m/s}^2$$

b)

$$a_x = \frac{dv_x}{dt} = -10t \text{ m/s}^2$$

$$a_x(t = 2s) = -20 \text{ m/s}^2$$