

Chapter 4

The special theory of relativity

Before the beginning of the twentieth century , two branches of physics (mechanics and electromagnetism) had developed quite independently of each other , the laws of mechanics and electromagnetism had been verified to such an extent that physicists were sure that there would be no further modification or improvements of them, but to every one's surprise ,early in this century ,physicist were faced with many new and basic problems, first they found that Newton's second law of motion which had been so well established for objects moving with low speeds ,did not give the correct results when applied to object moving with high speeds (speeds comparable to the speed of light) . Second, they found that for two observers in relative motion, one could not use the same set of transformation equations to transform the laws of mechanics and electromagnetism from the frame of reference of one observer to the frame of reference of the other observer. These and other difficulties were overcome by the formulation of the special theory of relativity by Einstein in 1905.

Galilean transformation

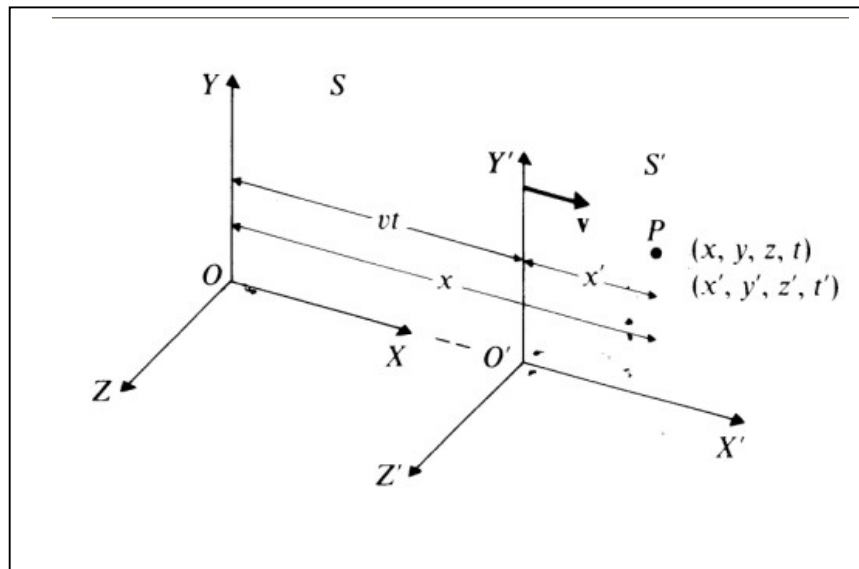
According to Newton first law, a system at rest will remain at rest or a system in uniform motion will remain in uniform motion if no net external forces act on the system.

Such systems in which the law of inertia holds are called inertial systems. For all practical purpose, a set of coordinates axes attached to the earth may be regarded as an inertial system, provided we neglect the small acceleration resulting from the rotational and orbital motion of the earth.

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Let us see how we transfer the coordinates of an event from one inertial system to another inertial system which is moving with a uniform velocity relative to the first. In Newtonian mechanics such transformation are made by the Galilean transformation equations.

Let one set of coordinates axis xyz be located in an inertial system S , set $x'y'z'$ in an inertial system which is moving with respect to system S with a velocity v along the xx' axes as shown in fig(1).



The origins of the two inertial systems coincide at $t=t'=0$. let the coordinates of an event taking place at some point P be (x,y,z,t) and (x',y',z',t') in the inertial systems respectively . According to fig (1). These coordinate are related by the Galilean coordinate transformation equations

$$\begin{array}{ll}
 x' = x - vt & x = x' + vt \\
 y' = y & y = y' \\
 z' = z & z = z' \quad \dots\dots\dots(1) \\
 t' = t & t = t'
 \end{array}$$

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Note : that in Newtonian relativity we always assume that $t' = t$

If we differentiate the above equations, assuming that $\frac{d}{dt}$ and $\frac{d}{dt'}$ are

identical we get the following velocity transformation equations

$$\begin{aligned} \frac{dx'}{dt'} &= \frac{dx}{dt} - v & u'_x &= u_x - v \\ \frac{dy'}{dt'} &= \frac{dy}{dt} & u'_y &= u_y & \dots\dots(2) \\ \frac{dz'}{dt'} &= \frac{dz}{dt} & u'_z &= u_z \end{aligned}$$

That is

Differentiating once again, we get acceleration transformation equations

$$\begin{aligned} \frac{d^2x'}{dt'^2} &= \frac{d^2x}{dt^2} & a'_x &= a_x \\ \frac{d^2y'}{dt'^2} &= \frac{d^2y}{dt^2} & a'_y &= a_y & \dots\dots\dots(3) \\ \frac{d^2z'}{dt'^2} &= \frac{d^2z}{dt^2} & a'_z &= a_z \end{aligned}$$

That is

That is, the acceleration is the same as viewed from either inertial system.

We can go step further and show that an equation which describes an event in reference frame does not change its form when transferred to another reference by using Galilean transformation for example. The components of a force F acting on a particle of mass m at a point P in reference system S may be written as

$$F_x = m \frac{d^2x}{dt^2} \quad , \quad F_y = m \frac{d^2y}{dt^2} \quad , \quad F_z = m \frac{d^2z}{dt^2}$$

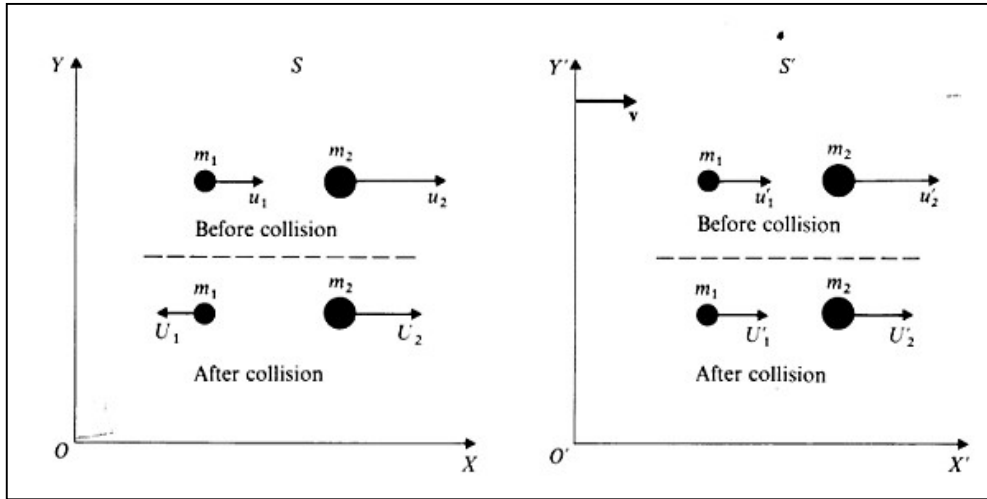
Using the values of xyz and t or directly from equation (3) we get

$$F'_x = m \frac{d^2x'}{dt'^2} \quad , \quad F'_y = m \frac{d^2y'}{dt'^2} \quad , \quad F'_z = m \frac{d^2z'}{dt'^2}$$

And this implies that the form of the equation has not changed under Galilean transformation (Newton's second law)

Example (1) illustrates the invariance of the form of the conservation of linear momentum and kinetic energy under Galilean transformation.

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Consider a collision between two masses, the conservation of linear momentum and kinetic energy may be written as

$$m_1 u_1 + m_2 u_2 = m_1 U_1 + m_2 U_2$$

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 U_1^2 + \frac{1}{2} m_2 U_2^2$$

Let us observe this collision from another inertial system S' moving with velocities u'_1 and u'_2 a long the x' -axis

$$m_1(u'_1 + v) + m_2(u'_2 + v) = m_1(U'_1 + v) + m_2(U'_2 + v)$$

This is on simplification

$$m_1 u'_1 + m_2 u'_2 = m_1 U'_1 + m_2 U'_2$$

Now

$$\frac{1}{2} m_1 (u'_1 + v)^2 + \frac{1}{2} m_2 (u'_2 + v)^2 = \frac{1}{2} m_1 (U'_1 + v)^2 + \frac{1}{2} m_2 (U'_2 + v)^2$$

By simplification we get

$$\frac{1}{2} m_1 u_1'^2 + \frac{1}{2} m_2 u_2'^2 = \frac{1}{2} m_1 U_1'^2 + \frac{1}{2} m_2 U_2'^2$$