Assist. Prof.: Ameen Abdulzahra

Lecture eighth: **Sum -of- product representation of logic function :**

A SP expression is a product term or several product terms, logically added together e.g:

$F = A.B + AB\overline{C} + BD + \ldots\ldots.$

↑ product (AND)

Derivation of sp :

1-construct the T.T.

2-construct a multiplication column of product of all inputs.

3-the desired expression is the sum of the product of all terms in which the output is

EX: For the following T.T. , write the logic function using sp method :

AB	Z		P terms
00	1	⇒	$\overline{A}\,\overline{B}$ ←
01	0		$\overline{A}B$
10	0		$A\overline{B}$
11	1		AB ←

$Z = \overline{A}\,\overline{B} + AB$

EX: For the following T.T. , write the logic function using sp method , then simplify it :

ABC	Z		P terms	min terms
000	0	⇒	$\overline{A}\,\overline{B}\,\overline{C}$	m_0
001	0		$\overline{A}\,\overline{B}C$	m_1
010	0		$\overline{A}B\overline{C}$	m_2
011	1		$\overline{A}BC$	m_3 ←
100	0		$A\overline{B}\,\overline{C}$	m_4
101	1		$A\overline{B}C$	m_5 ←
110	1		$AB\overline{C}$	m_6 ←
111	1		ABC	m_7 ←

$Z = m_3 + m_5 + m_6 + m_7$

$= \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$

$= BC(\overline{A} + A) + A\overline{B}C + AB\overline{C}$

$= BC + A\overline{B}C + AB\overline{C}$

$= C(B+\overline{B}A) + AB\overline{C} = C(B+A) + AB\overline{C}$

$= CB + CA + AB\overline{C} = CB + A(C+B\overline{C})$

$= CB + A(C+B)$

$= CB + AC + AB$

Product -of- sum representation of logic function :

A PS is a sum term or several sum terms logically multiplied together e.g. :

$F = (A+B)(\bar{A}+\bar{B}+C)(A+D)\ldots\ldots$

Derivation of PS :

1-construct the T.T.

2-construct a sum column of sum of all inputs (0=uncomplement , 1=complement)

3-The desired output exp. Is the product of the sum of all terms in which the output is zero.

EX: For the following T.T. , write the logic function using PS method :

AB	Z
00	1
01	0
10	0
11	0

⟹

S. treams	Max terms
$(A+B)$	M_0
$(A+\bar{B})$	M_1 ←
$(\bar{A}+B)$	M_2 ←
$(\bar{A}+\bar{B})$	M_3 ←

$Z = M_1 . M_2 . M_3$

$= (A+\bar{B})(\bar{A}+B)(\bar{A}+\bar{B})$

EX: Simplify the following function using SP and PS methods :

$F(A,B,C) = \pi(M_2, M_3, M_6)$

Sol:

ABC	Z	
000	1	m_0
001	1	m_1
010	0	M_2
011	0	M_3
100	1	m_4
101	1	m_5
110	0	M_6
111	1	m_7

1-By SP method :

$Z = m_0 + m_1 + m_4 + m_5 + m_7$

$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + ABC$

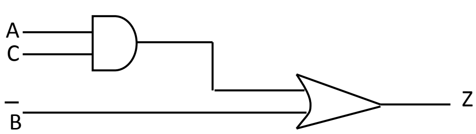
$= \bar{A}\bar{B}(\bar{C}+C) + A\bar{B}(\bar{C}+C) + ABC$

$= \bar{A}\bar{B} + A\bar{B} + ABC$

$= \bar{B}(\bar{A}+A) + ABC$

$= \bar{B} + BAC$

$Z = \bar{B} + AC$



2- By PS method :

$Z = M_2 . M_3 . M_6$

$=(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$

$=(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)(A+\bar{B}+C)$

$=(AA+\bar{B}A+CA+A\bar{B}+\bar{B}\bar{B}+C\bar{B}+A\bar{C}+\bar{B}\bar{C}+C\bar{C})$

$=(A+\bar{B}A+CA+A\bar{B}+\bar{B}+C\bar{B}+A\bar{C}+\bar{B}\bar{C}).($

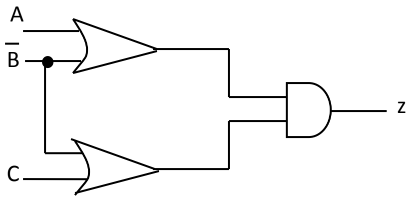
$=(A(1+\bar{B}+C+\bar{B}+\bar{C})+\bar{B}(1+C+\bar{C})).($

$=(A+\bar{B})(\bar{A}A+\bar{B}A+CA+\bar{A}\bar{B}+\bar{B}\bar{B}+C\bar{B}+\bar{A}C+\bar{B}C+CC)$

$=(A+\bar{B})(\bar{B}A+CA+\bar{A}\bar{B}+\bar{B}+C\bar{B}+\bar{A}C+\bar{B}C+C)$

$=(A+\bar{B})(\bar{B}(A+\bar{A}+1+C+C)+C(A+\bar{A}+1))$

$=(A+\bar{B})(\bar{B}+C)$



PS method require one more gate than