

Composition of Functions

Def:-

Given Functions f and g , the composition of f and g is denoted by $f \circ g$ is the function defined by

$$(f \circ g)(x) = f(g(x))$$

$$D_{f \circ g} = \{x = g(x) \in D_f, x \in D_g\}$$

Example 1

Find Formula For

1- $(f \circ g)(x)$

2- $(g \circ f)(x)$

3- $g(f(2))$

if $f(x) = x^2$, $g(x) = x - 7$.

Sol:-

$$\begin{aligned} 1- (f \circ g)(x) &= f(g(x)) \\ &= f(x - 7) \\ &= (x - 7)^2 \end{aligned}$$

$$\begin{aligned} 2- (g \circ f)(x) &= g(f(x)) \\ &= g(x^2) = x^2 - 7 \end{aligned}$$

$$\begin{aligned} 3- (g \circ f)(2) &= g(f(2)) = g(x^2) \\ &= g(2^2) \\ &= 4 - 7 = -3. \end{aligned}$$

Example 2 :- $f(x) = 2x + 1$, $f: \mathbb{N} \rightarrow \mathbb{N}$

$g(x) = x^2$, $g: \mathbb{N} \rightarrow \mathbb{N}$

Find the value of $f \circ g$?

$$(f \circ g)(x) = 33$$

Solution:-

$$(f \circ g)(x) = 33$$

$$f(g(x)) = 33$$

$$f(x^2) = 33$$

$$2x^2 + 1 = 33 \Rightarrow 2x^2 = 32$$

$$x^2 = 16$$

$$x = 4$$

Ex 3

$$f(x) = \frac{x}{2}$$

$$g(x) = \sin x$$

is it true $(f \circ g)(x) = (g \circ f)(x)$??

Solution:- $(f \circ g)(x) = f(\sin x) = \frac{\sin x}{2}$

$$(g \circ f)(x) = g\left(\frac{x}{2}\right) = \sin\left(\frac{x}{2}\right)$$

it is not true.

Domain of Composition of Functions

Example:- Find Domain of $(F \circ g)(x)$?

$$F(x) = x^2, \quad g(x) = x - 7$$

Solution:-

$$(F \circ g)(x) = F(g(x))$$

inside Function $\rightarrow g(x)$

$$g(x) = x - 7$$

$\rightarrow$ Polynomial Fun. $F: \mathbb{R} \rightarrow \mathbb{R}$

1- $\therefore D_g = \mathbb{R}$

2- $(F \circ g)(x) = (x - 7)^2$

$\rightarrow$ Polynomial $F: \mathbb{R} \rightarrow \mathbb{R}$

$$\therefore D_{(F \circ g)} = \mathbb{R}$$

$$\Rightarrow D_{F \circ g} \cap D_g = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$$

Ex 2:- Find Domain of $(F \circ g)(x)$?

$$F(x) = x^2 + 1, \quad g(x) = \sqrt{x - 2}$$

Sol:- $g(x) = \sqrt{x - 2} \leftarrow \begin{matrix} \text{بالقيمة} \\ \text{غير السالبة} \end{matrix} \quad \begin{matrix} x - 2 \geq 0 \\ \rightarrow x \geq 2 \end{matrix}$

$$\therefore D_g = x \geq 2$$

$$(F \circ g)(x) = x - 1$$

$\rightarrow$ Polynomial Fun.

$$\therefore D_{F \circ g} = \mathbb{R}$$

$$\Rightarrow D_{F \circ g} \cap D_g = \mathbb{R} \cap x \geq 2 = x \geq 2$$