Chapter three

Numerical Solution of the linear equations System

الحلول العددية لمنظومة المعادلات الخطية

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots \qquad \qquad \vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \qquad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$ → square Matrix

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$$ → diagonal Matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ → Identity Matrix

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

مصفوفة مثلثية
سفلى
lower triangular

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

upper triangular
مصفوفة مثلثية عليا

Jacobi's Method

1- $x_i = \frac{1}{a_{ii}} \left(b_i - \sum_{i=1}^{n} a_{ij} x_j \right)$ ← صيغة جاكوبي

$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{i=1}^{n} a_{ij} x_j^{(k)} \right)$ ← الصيغة التكرارية لجاكوبي

2- $\max \sum_{\substack{j=1 \\ j \neq i}}^{n} \left| \frac{a_{ij}}{a_{ii}} \right| < 1$ ← شرط التقارب

3- $\left| x_i^{(k+1)} - x_i^{(k)} \right| \leq \epsilon$ ← شرط التوقف

Ex:- By using the Jacobi Method. Find the solution of the following Systems?

$$4x_1 + x_2 - 2x_3 = 1$$
$$x_1 - 7x_2 + 10x_3 = 2$$
$$x_1 + 3x_2 - x_3 = 8$$

$x_1^{(0)} = 1$, $x_2^{(0)} = 3$, $x_3^{(0)} = 2$, Use three loops.

Solution:-

$$\max \sum_{\substack{j=1 \\ j \neq i}}^{n} \left| \frac{a_{ij}}{a_{ii}} \right| < 1$$

$\underset{a_{11}}{\boxed{4}} x_1 + x_2 - 2x_3 = 1 \rightarrow \left| \frac{1}{4} \right| + \left| \frac{-2}{4} \right| = 0.25 + 0.5$

$= 0.75$

(2)

$x_1 - \boxed{7} x_2 + 10 x_3 = 2 \rightarrow \left|\frac{1}{-7}\right| + \left|\frac{10}{-7}\right| = 0.143 + 1.429$

$= 1.572$

a_{22}

$x_1 + 3x_2 \cancel{+} x_3 = 8 \rightarrow \left|\frac{1}{-1}\right| + \left|\frac{3}{-1}\right| = 1 + 3 = 4$

$1 = a_{33}$

$0.75 \ , \ 1.572 \ , \ 4 \qquad \therefore Max = 4 \not< 1$

Max

is unconvergence

∴ يعاد الترتيب يجب ان يكون القطر a_{11} ، a_{22} ، a_{33} في كل معادلة

هو اكبر بالحد الخاص بي

$\boxed{4} x_1 + x_2 - 2x_3 = 1$

$x_1 + \boxed{3} x_2 - x_3 = 8$

$x_1 - 7x_2 + \boxed{10} x_3 = 2$

نستخدم الصيغة الخاصة بطريقة Jacobi

$$x_i^{(K+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{i=1}^{n} a_{ij} x_j^{(K)} \right)$$

$K=0 \quad , x_1^{(0)} = 1 , \quad x_2^{(0)} = 3 , \quad x_3^{(0)} = 2$

$$x_1^{(1)} = \frac{1}{4} \left(1 - x_2^{(0)} + 2x_3^{(0)} \right) = \frac{1}{4} (1 - 3 + 4) = \frac{1}{2} = 0.5$$

$$x_2^{(1)} = \frac{1}{3} (8 - 1 + 2) = \frac{9}{3} = 3$$

$$x_3^{(1)} = \frac{1}{10} (2 - 1 + 21) = \frac{22}{10} = 2.2$$

$\therefore x_1^{(1)} = 0.5 \quad , x_2^{(1)} = 3 \quad , x_3^{(1)} = 2.2$

$$x^{(1)} = \begin{bmatrix} 0.5 \\ 3 \\ 2.2 \end{bmatrix}$$

(3)

«Jacobi Method»

$k=1$

$$x_1^{(2)} = \frac{1}{4}(1-3+2(2.2)) = 0.6$$

$$x_2^{(2)} = \frac{1}{3}(8-0.5+2.2) = 3.233$$

$$x_3^{(2)} = \frac{1}{10}(2-0.5+21) = 2.25$$

$$x^{(2)} = \begin{bmatrix} 0.6 \\ 3.233 \\ 2.25 \end{bmatrix}$$

$k=2$

$$x_1^{(3)} = \frac{1}{4}(1-3.233+2(2.25)) = 0.56$$

$$x_2^{(3)} = \frac{1}{3}(8-0.6+2.25) = 3.217$$

$$x_3^{(3)} = \frac{1}{10}(2-0.6+7(3.233)) = 2.403$$

$$x^{(3)} = \begin{bmatrix} 0.56 \\ 3.217 \\ 2.403 \end{bmatrix}$$