التفاضل والتكامل المتقدم

المرحلة الثانية

قسم الرياضيات

§1 : Functions of Two or More Variables

Definition 1.1: Suppose D is a collection of n-tuples of real numbers $(x_1, x_2, \ldots, x_n)$. A function f with domain D is a rule that assigns a number $w = f(x_1, x_2, \ldots, x_n)$ to each n-tuple in D. The set of all values $w = f(x_1, x_2, \ldots, x_n)$ is called the range of f. The symbol w is called the dependent variable of f, and f is said to be a function of the n independent variables $x_1, x_2, \ldots, x_n$.

Example 1.2: In the function $V = \pi r^2 h$, the dependent variable is V, and the independent variables are r and h.

Remark 1.3: Functions given by formulas are evaluated in the usual way by substituting values of the independent variables and calculating the corresponding value of the dependent variable. As example the value of V in example 1.2 when $r = 2$ and $h = 3$ is 12π.

Examples 1.4: 1. The value of the function $w = f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the point $(3, 12, 4)$ is $\sqrt{3^2 + 12^2 + 4^2} = \sqrt{9 + 144 + 16} = \sqrt{169} = 13$.

2. The value of the function $z = x^2 + y^2$ at the point $(2, 3)$ is $2^2 + 3^2 = 4 + 9 = 13$.

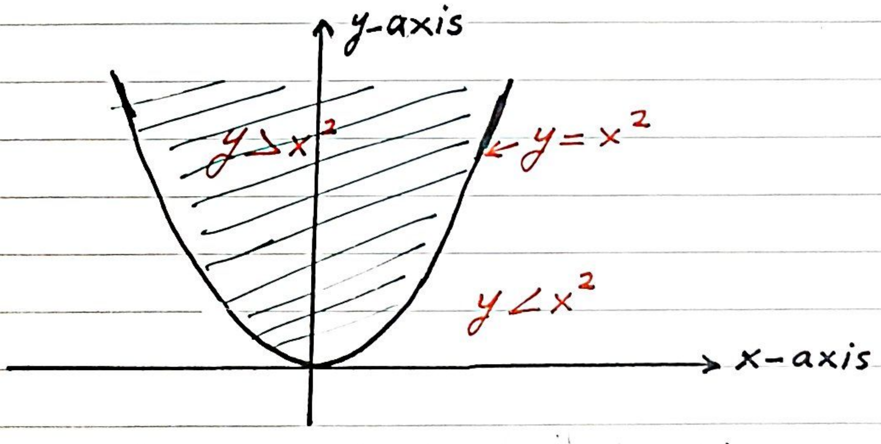
Example 1.5: Sketch the domain of the function $f(x,y)=\sqrt{y-x^2}$. What is the range of the function f?

Solution: The domain D_f is the set of all pairs (x,y) in the xy-plane for which $\sqrt{y-x^2}$ is real which are the points (x,y) which satisfy $y-x^2 \geq 0$ (i.e. $y \geq x^2$).

Thus $D_f = \{(x,y \in \mathbb{R}^2 : y-x^2 \geq 0\}$

$$= \{(x,y) \in \mathbb{R}^2 : y \geq x^2\}$$

Therefore D_f is the set of all points that lies above and on the parabola $y=x^2$,



and R_f is the set of all non-negative numbers $z \geq 0$ where $z=\sqrt{y-x^2}$.

Example 1.6: What are the domain and the range of the function $f(x,y)=\dfrac{xy}{x^2-y^2}$?

Solution: The function $z=f(x,y)$ is defined for all the values of x and y except the values which make the denominator x^2-y^2 is equal to zero. $x^2-y^2=0 \Rightarrow x^2=y^2 \Rightarrow y=\pm x$.

Therefore $D_f = \{(x,y) \in \mathbb{R}^2 : y \neq x \text{ and } y \neq -x\}$, and

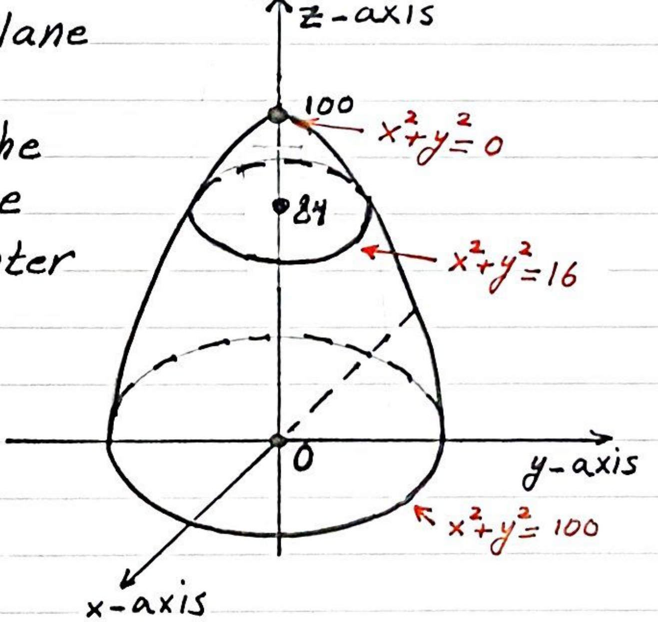
$R_f = \{z : -\infty < z < \infty\} = \{z : z \in \mathbb{R}\} = \mathbb{R}$.

Example 1.7: Analyze and graph the function $z = f(x,y) = 100 - x^2 - y^2$ and describe and plot the level curves $f(x,y) = 0, 84, 100$.

Solution: The graph of f is a circular paraboloid intercepting the z-axis at the point $(0,0,100)$.

$D_f = \{(x,y) : (x,y) \in \mathbb{R}^2\} = \mathbb{R}^2$.

$R_f = \{z \in \mathbb{R} : z \leq 100\}$.



1) $f(x,y)=0$ is the set of all the points in the xy-plane at which $100-x^2-y^2=0$ $\Rightarrow x^2+y^2=100$ which is the circle in the xy-plane of radius 10 and center $(0,0,0)$.

2) $f(x,y)=84$ is the set of all the points in the plane $z=84$ at which $100-x^2-y^2=84 \Rightarrow x^2+y^2=16$ which is a circle in the plane $z=84$ of radius 4 and center $(0,0,84)$.

3) $f(x,y)=100$ is the point $(0,0,100)$ in the level plane $z=100$.