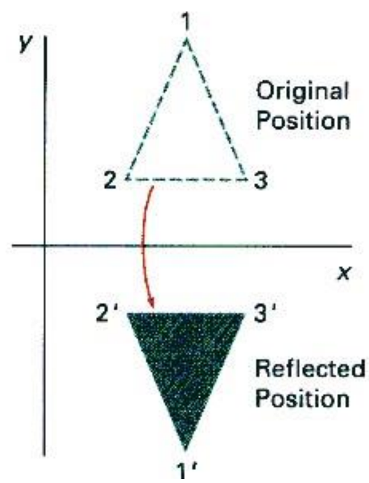
## Reflection

- Transformation that produces a mirror image of an object is called *reflection*.
- Image is generated relative to an axis of reflection by rotating the object **180°** about the reflection axis.
- A Common reflections as follows:

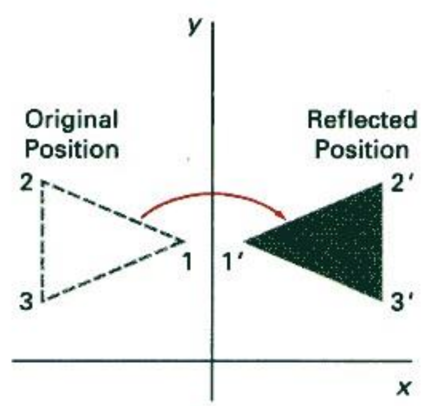
1- Reflection about the line y=0 (the x-axis) is accomplished with the transformation matrix:

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$ → Reflection in the x-axis



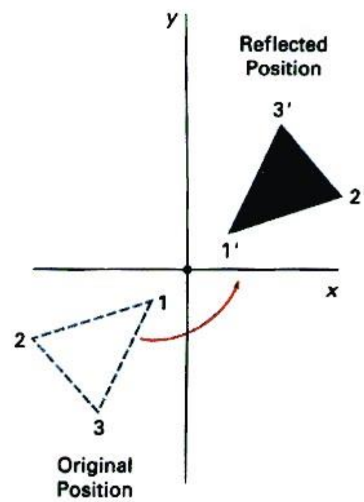

2- Reflection about the line x=0 (the y-axis) is accomplished with the transformation matrix:

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$ → Reflection in the y-axis



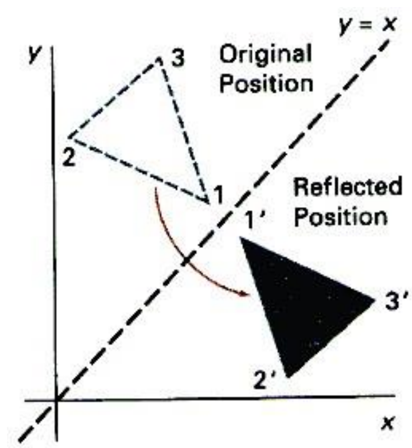
3- Reflection about the origin which is equivalent to the rotation matrix R($\theta$) with $\theta$=180°

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ → Reflection about the origin



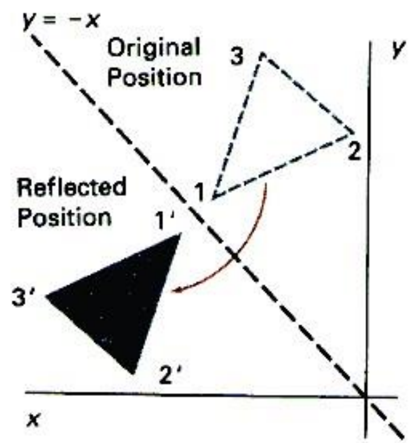

4- Reflection in the line y=x (diagonal line)

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ → Reflection about the diagonal line y=x



5- Reflection about the diagonal line y=-x

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$ → Reflection about the diagonal line y=-x




**Coordinate plane rules:**

Over the x-axis : (x, y) → (x, −y)

Over the y-axis : (x, y) → (−x, y)

Through the origin : (x, y) → (−x, −y)

Over the line y = x : (x, y) → (y, x)

Over the line y = -x : (x, y) → (-y, -x)

Example1: Given a triangle with coordinate points A(3, 4), B(6, 4) and C(5, 6). Apply the reflection transformation:

1- about the x-axis

2- about the y-axis

3- about the origin

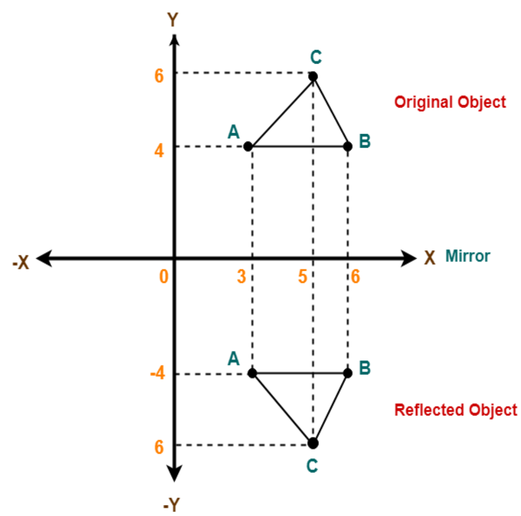
4- about the diagonal line y=x

5- about the diagonal line y=-x

Sol: \ 1- about the x-axis

$$y=0 \quad ; \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \rightarrow \text{ reflection matrix}$$

$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 6 & -4 \\ 5 & -6 \end{bmatrix}$$




OR

Sol: \ 1- about the x-axis

$$y=0 \quad ; \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \rightarrow \text{ reflection matrix}$$

$$[3 \;\; 4] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = [3 \; -4]$$

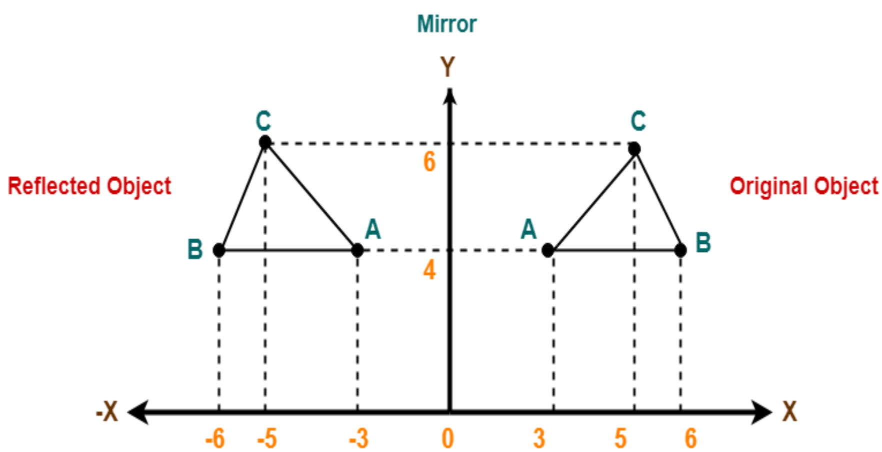
$$[6 \;\; 4] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = [6 \; -4]$$

$$[5 \;\; 6] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = [5 \; -6]$$

Sol: \ 2- about the y-axis

x=0 ; $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow$ reflection matrix

$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ -6 & 4 \\ -5 & 6 \end{bmatrix}$$




Sol: \ 3- about the origin

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \rightarrow$ reflection matrix

$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ -6 & -4 \\ -5 & -6 \end{bmatrix}$$

Sol: \ 4- about the diagonal line y=x

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow \text{ reflection matrix}$$

$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}=\begin{bmatrix} 4 & 3 \\ 4 & 6 \\ 6 & 5 \end{bmatrix}$$

Sol: \ 5- about the diagonal line y=-x

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \rightarrow \text{ reflection matrix}$$

$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}=\begin{bmatrix} -4 & -3 \\ -4 & -6 \\ -6 & -5 \end{bmatrix}$$

## Shear

- Distorting or changing the shape of an object by differentially moving some of its vertices as if the object internal layers are sided over each other is called **Shear.**
- Shears either shift coordinates x values or y values
- Similar to scaling, the shear transformation requires two parameters ($s_x$, $s_y$) not on the main diagonal of the transformation matrix but on the other two positions.

$$\begin{bmatrix} 1 & s_x \\ s_y & 1 \end{bmatrix}$$

- Applying a shear transformation sh($s_x$, $s_y$) to point (x,y) yields the point ($\bar{x}$, $\bar{y}$) with the new coordinates.

$$[\bar{x} \quad \bar{y}] = [x \quad y]\begin{bmatrix} 1 & s_x \\ s_y & 1 \end{bmatrix} = [x + y * s_y \qquad x * s_x + y]$$

- Shear transformations are carried out with respect to the origin of the coordinate system so that an object that is not centered around the origin will not only be deformed by a shear transformation but also shifted.
- If $s_x$=0 the shear take place in the x direction and in the y direction if $s_y$=0

Example: Consider the square whose vertices are: A(0,0), B(1,0), C(1,1), and D(0,1). Perform the shear transformation:

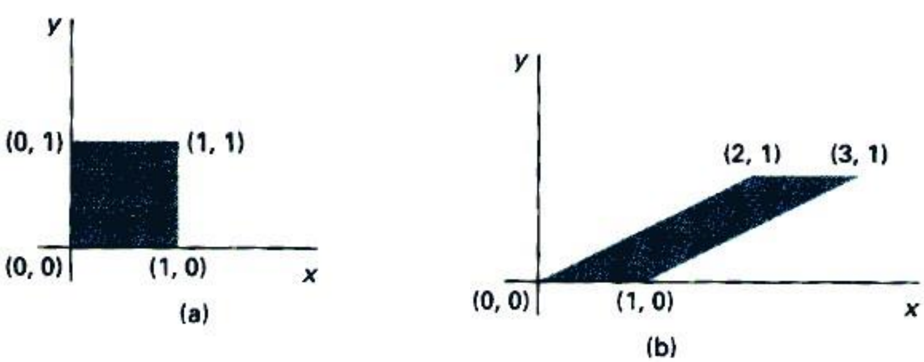
1- In the x direction 2-
In the y direction
3- In both direction
Where $s_x$=1 and $s_y$=2

**Sol:**

1- Shear in the x direction:

$s_x$=0 ; $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ → shear matrix

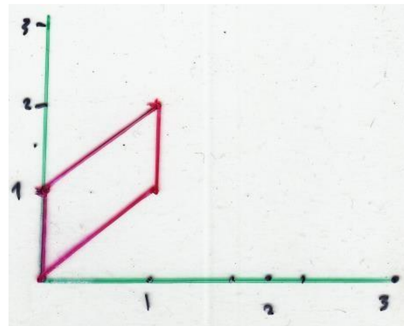
$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 1 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$



## 2- Shear in the y direction:

$s_y$=0

$\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ → shear matrix



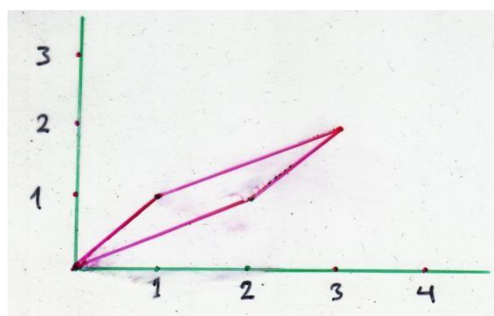
$$\begin{bmatrix}0 & 0\\0 & 1\\1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 1\\1 & 1\\1 & 2\end{bmatrix}$$

## 3- Shear in both direction:

$s_x$=1 $s_y$=2

$\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$ → shear matrix

$$\begin{bmatrix}0 & 0\\0 & 1\\1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}=\begin{bmatrix}0 & 0\\2 & 1\\1 & 1\\3 & 2\end{bmatrix}$$




**H.W** Perform a shear transformation for the shape A(3,0), B(3,3), C(5,3), and D(0,5)

1- In the x-direction
2- In the y-direction
3- In the both direction
   Where $s_x$=2 $s_y$=1