Newton Divided Difference Interpolation Polynomial (NDDIP) for equal spacing

This method is used when the difference of the given values of the variable x is a fixed constant h (i.e. the given values of x are $x_0, x_1, \ldots, x_n$ where $x_1 - x_0 = x_2 - x_1 = \ldots = x_n - x_{n-1} = h$).
This method is also called Newton–Gregory Interpolation Formula.

تستخدم هذه الطريقة عندما يكون الفرق في القيم المعطاة للمتغير x مقدار ثابت (أي أن القيم المعطاة للمتغير x هي $x_0, x_1, \ldots, x_n$ بحيث أن $x_2 - x_1 = x_1 - x_0 = \ldots = x_n - x_{n-1} = h$).

تسمى هذه الطريقة ايضا بصيغة نيوتن-جريجوري للاستكمال.

There are two ways to interpolate by using this method which are the Newton forward difference and the Newton backward difference depending on the position of x with respect to the given data.

Newton Forward-Difference Interpolation

This method is used when the interpolation is to be calculated for a value of x in the first half of the given data

تستخدم هذه الطريقة عندما يطلب احتساب الاستكمال لقيم x التي تقع في النصف الأول من البيانات المعطاة.

For a given $n+1$ values $x_0, x_1, \ldots, x_n$ of x the polynomial $P_n(x)$ will be given in the following formula;

لـ $n+1$ من القيم المعطاة $x_0, x_1, \ldots, x_n$ للمتغير x تكون متعددة الحدود $P_n(x)$ بالصيغة التالية:

$$P_n(x) = y_0 + k \cdot \frac{\Delta y_0}{1!} + k(k-1) \cdot \frac{\Delta^2 y_0}{2!}$$

$$+ k(k-1)(k-2) \cdot \frac{\Delta^3 y_0}{3!} + \cdots$$

$$+ k(k-1)\cdots(k-n+1) \cdot \frac{\Delta^n y_0}{n!} .$$

where $k = \dfrac{x - x_0}{h}$, $h = x_1 - x_0 = x_2 - x_1 = \cdots = x_n - x_{n-1}$,

and Δ is the forward difference operator and the values of $\Delta y_0, \Delta^2 y_0, \ldots, \Delta^n y_0$ can be calculated by using the forward difference table.

The Forward Difference table

If we have 5 given points $(x_0, y_0), (x_1, y_1), \ldots, (x_4, y_4)$ for example then $n = 4$ and the forward difference table will be :

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$
x_0	y_0				
		$\Delta y_0 = y_1 - y_0$			
x_1	y_1		$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$		
		$\Delta y_1 = y_2 - y_1$		$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$	
x_2	y_2		$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$		$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$
		$\Delta y_2 = y_3 - y_2$		$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$	
x_3	y_3		$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$		
		$\Delta y_3 = y_4 - y_3$			
x_4	y_4				

Example 1: By using NDDIP, find y at $x = 7$ from the following data

x	1	6	11	16	21	26
y	7	12	16	21	28	39

Solution:

Since $x=7$ is in the first half, then we will use the forward difference interpolation as follows:

$h = x_{i+1} - x_i = 5$,

$$k = \frac{x - x_0}{h} = \frac{7-1}{5} = \frac{6}{5} = 1.2$$

The forward difference table will be

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$	$\Delta^5 y_i$
		Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$
$x_0=1$	7					
		5				
$x_1=6$	12		-1			
		4				
$x_2=11$	16		1	2		
		5			-1	
$x_3=16$	21		2	1		-1
		7			-2	
$x_4=21$	28		1	-1		
		8				
$x_5=26$	36					

$$P_5(7) = y_0 + k\,\Delta y_0 + k(k-1)\frac{\Delta^2 y_0}{2!}$$

$$+ k(k-1)(k-2)\frac{\Delta^3 y_0}{3!} + k(k-1)(k-2)(k-3)\frac{\Delta^4 y_0}{4!}$$

$$+ k(k-1)(k-2)(k-3)(k-4)\frac{\Delta^5 y_0}{5!}$$

$$= 7 + (1.2)*(5) + (1.2)*(1.2-1)*\left(\frac{-1}{2!}\right)$$

$+(1.2)*(1.2-1)*(1.2-2)*(\frac{2}{3!})$

$+(1.2)*(1.2-1)*(1.2-2)*(1.2-3)*(\frac{-1}{4!})$

$+(1.2)*(1.2-1)*(1.2-2)*(1.2-3)*(1.2-4)*(\frac{-1}{5!})$

$=7+(1.2)*(5)+(1.2)*(0.2)*(\frac{-1}{2})$

$+(1.2)*(0.2)*(-0.8)*(\frac{2}{6})$

$+(1.2)*(0.2)*(-0.8)*(-1.8)*(\frac{-1}{24})$

$+(1.2)*(0.2)*(-0.8)*(-1.8)*(-2.8)*(\frac{-1}{120})$

$=12.809664.$

Therefore the interpolated point is

$(x,y)=(7,12.809664)$, i.e. $x=7$

and $y=12.809664$.

Example 2: By using NDDIP, find y at $x=0.5$ from the following data

x	0	1	2	3
y	0	1	8	27

Solution:

Since $x = 0.5$ is in the first half, then we will use the forward difference interpolation as follows:

$h = x_{i+1} - x_i = 1$,

$$k = \frac{x - x_0}{h} = \frac{0.5 - 0}{1} = 0.5$$

The forward difference table will be

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
0	0	Δy_0		
		1	$\Delta^2 y_0$	$\Delta^3 y_0$
1	1		6	
		7		6
2	8		12	
		19		
3	27			

$$P_3(0.5) = y_0 + k\,\Delta y_0 + k(k-1)\frac{\Delta^2 y_0}{2!}$$

$$+ k(k-1)(k-2)\frac{\Delta^3 y_0}{3!}$$

$$= 0 + (0.5) * 1 + (0.5) * (0.5-1) * \frac{6}{2!}$$

$$+ (0.5) * (0.5-1) * (0.5-2) * \left(\frac{6}{3!}\right)$$

$$= 0.5 + (0.5) * (-0.5) * \frac{6}{2}$$

$$+ (0.5) * (-0.5) * (-1.5) * \left(\frac{6}{6}\right) = 0.125$$

Therefore the interpolated point is $(x,y)=(0.5, 0.125)$, i.e. $x=0.5$, $y=0.125$.

Exercise: By using NDDIP, find y at $x=8$ from the following data:

x	0	5	10	15	20	25
y	7	11	14	18	24	32