5) The false position method:

This method is similar to the bisection method and requires two initial values a and b.

Algorithm Steps

1) Choose an interval $[a,b]$ such that $f(a) * f(b) < 0$

2) Find x_i as an instantaneous root:

$$x_i = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

3) Find $f(x_i)$

4) If $f(a) * f(x_i) < 0$, then $b = x_i$ and $f(b) = f(x_i)$.
If $f(a) * f(x_i) > 0$, then $a = x_i$ and $f(a) = f(x_i)$.

5) Repeat the steps (2) to (4) above to find a new x_i ... and so on.

6) End the calculations when the given accuracy condition is satisfied.

Example: Find the root of the equation $e^x - 5x = 0$ in $I = [2,3]$, correct to $\epsilon = 0.05$, using the false position method.

Solution:

Let $f(x) = e^{x} - 5x$, $a = 2$ and $b = 3$.

Since $f(2) = e^{2} - 10 = -2.610943901$ (-ve)

and $f(3) = e^{3} - 15 = 5.085536923$ (+ve)

, then $f(2) * f(3) < 0$ and a root of $f(x) = 0$ lies between 2 and 3.

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{2 f(3) - 3 f(2)}{f(3) - f(2)}$$

$$= 2.339238668$$

$f(x_1) = e^{x_1} - 5x_1 = -1.322857339$ (-ve)

Since $f(a) * f(x_1) > 0$, then $a = x_1$ and $f(a) = f(x_1) = -1.322857339$.

$$x_2 = \frac{2.339238668 f(3) - 3 f(2.339238668)}{f(3) - f(2.339238668)}$$

$$= 2.475636795$$

$f(x_2) = e^{x_2} - 5x_2 = -0.488908237$ (-ve)

Since $f(a) * f(x_2) > 0$, then $a = x_2$ and $f(a) = f(x_2) = -0.488908237$

$$x_3 = \frac{2.475636795 f(3) - 3 f(2.475636795)}{f(3) - f(2.475636795)}$$

$= 2.521626213$

$f(x_3) = e^{x_3} - 5x_3 = -0.15930641$ $(-ve)$

Since $f(a) * f(x_3) > 0$, then $a = x_3$ and $f(a) = f(x_3) = -0.15930641$.

$$x_4 = \frac{2.521626213\, f(3) - 3\, f(2.521626213)}{f(3) - f(2.521626213)}$$

$= 2.536156296$

$f(x_4) = e^{x_4} - 5x_4 = -0.049753863$

Since $|f(x_4)| = 0.049753863 < \epsilon$, then the root $\bar{x} = 2.536156296$

Exercises: Solve the following equations by using the false position method:

1) $x^3 - 2x^2 = x + 1$ in the interval $[-1, 0]$ and $\epsilon = 0.001$.
2) $e^x = 3x$ in the interval $[1, 2]$ and $\epsilon = 0.01$.