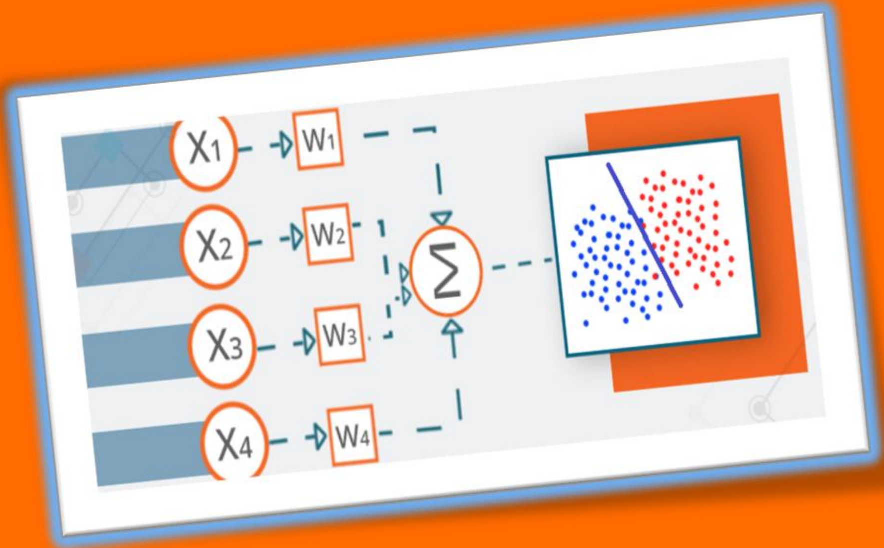
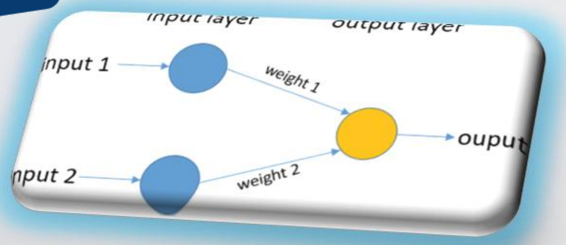


Single Layer Perceptron

Part 3



Prepared by Iman Hussein

Example

Train a perceptron network to simulate of "OR GATE",

Learning rate $\eta = 1$, $\theta = 0$,

Initial weights: $w_1 = 0$, $w_2 = 0$

Inputs		Goal outputs
X_1	X_2	$d = O_{desired}$
0	0	0
0	1	1
1	0	1
1	1	1

Solution:

X: input vector

u, vq, net : weighted sum

w : weight

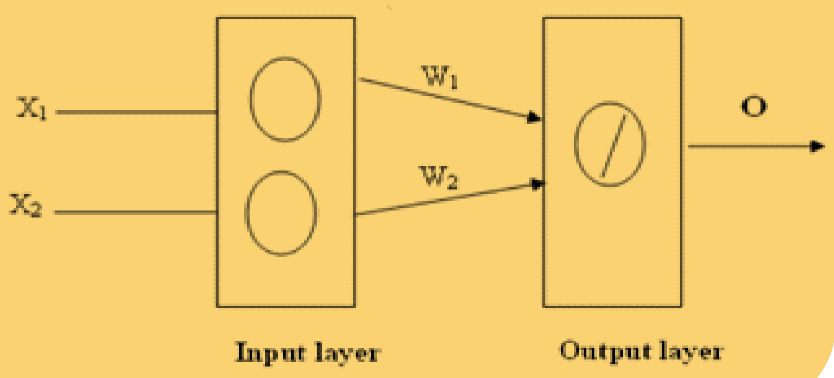
Δw: the weight change

η: learning rate

d : desired output

y= O : actual output

△= E : error between d and y



Input 1

When (X_1=0, X_2=0)

X_1	X_2	W1 old	W2 old	d=O desired	y = O actual	E	W1 new	W2 new
0	0	0	0	0				
0	1			1				
1	0			1				
1	1			1				

$u = \sum W_{ij} * X_j$ (weighted sum)

$O = f(\sum W_{ij} * X_j)$

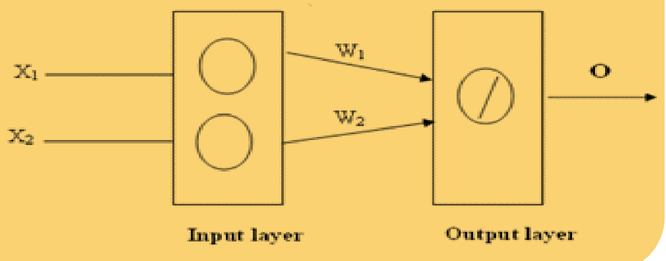
$= f(W_1 * X_1 + W_2 * X_2)$

$= f(0 * 0 + 0 * 0)$

$= f(0)$ $u <= \theta$, $(0 <= 0)$

$= 0$

$$y_j = f(net_j) = \begin{cases} 1 & \text{if } net_j > \theta \\ 0 & \text{if } net_j \le \theta \end{cases} \quad \text{where} \quad net_j = \sum_{i=1}^{n} x_i w_{ij}$$



Error signal= $\triangle i$ = **E = ($O_{desired}$ - O_{actual})**

E = (d - y)

E= (0-0) = 0

X_1	X_2	W1 old	W2 old	d=O desired	y = O actual	E	W1 new	W2 new
0	0	0	0	0	0	0		
0	1			1				
1	0			1				
1	1			1				

Input 1

When (X_1=0, X_2=0)

X_1	X_2	W1 old	W2 old	d=Odesired	y = Oactual	E	W1 new	W2 new
0	0	0	0	0	0	0		
0	1			1				
1	0			1				
1	1			1				

Adjust the weights = update of weights

Wij new = Wij old + △Wij

△Wij=η △i X_j

Wij new = Wij old + η △i X_j

Wij new = Wij old + η E X_j

W1 new = W1old + η E X1

W1 new = 0 + 1* 0 * 0

W1 new = 0

W2 new = W2old + η E X2

W2 new = 0 + 1* 0 * 0

W2new = 0

X_1	X_2	W1 old	W2 old	d=Odesired	y = Oactual	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1			1				
1	0			1				
1	1			1				

X_1	X_2	W1 old	W2 old	d=Odesired	y = Oactual	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1				
1	0			1				
1	1			1				

Input ②

When (X_1=0, X_2=1)

X_1	X_2	W1 old	W2 old	d=O desired	y = O actual	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1				
1	0			1				
1	1			1				

$u = \sum W_{ij} * X_j$ (weighted sum)

$O = f(\sum W_{ij} * X_j)$

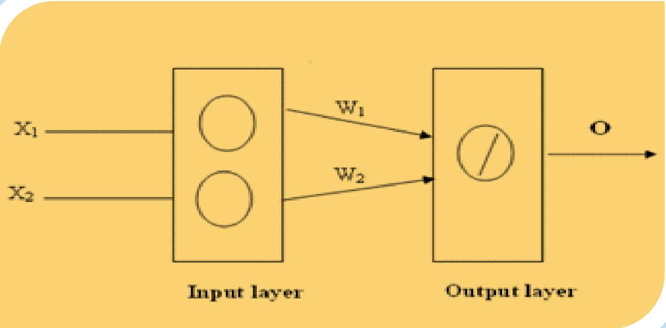
$= f(W_1 * X_1 + W_2 * X_2)$

$= f(0 * 0 + 0 * 1)$

$= f(0) \quad u <= \theta, \quad (0 <= 0)$

$= 0$

$$y_j = f(net_j) = \begin{cases} 1 & \text{if } net_j > \theta \\ 0 & \text{if } net_j \le \theta \end{cases} \quad \text{where} \quad net_j = \sum_{i=1}^{n} x_i w_{ij}$$



Error signal= Δi = **E = ($O_{desired}$ - O_{actual})**

E = (d - y)

E= (1-0) = 1

X_1	X_2	W1 old	W2 old	d=O desired	y = O actual	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1		
1	0			1				
1	1			1				

Input 2

When (X_1=0, X_2=1)

X_1	X_2	W1 old	W2 old	d=Odesired	y = Oactual	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1		
1	0			1				
1	1			1				

Adjust the weights = update of weights

$W_{ij\ new} = W_{ij\ old} + \triangle W_{ij}$

$\triangle W_{ij} = \eta\ \triangle_i\ X_j$

$W_{ij\ new} = W_{ij\ old} + \eta\ \triangle_i\ X_j$

$W_{ij\ new} = W_{ij\ old} + \eta\ E\ X_j$

$W_{1\ new} = W_{1old} + \eta\ E\ X_1$

$W_{1\ new} = 0 + 1 * 1 * 0$

$W_{1\ new} = 0$

$W_{2\ new} = W_{2old} + \eta\ E\ X_2$

$W_{2\ new} = 0 + 1 * 1 * 1$

$W_{2new} = 1$

X_1	X_2	W1 old	W2 old	d=Odesired	y = Oactual	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1	0	1
1	0			1				
1	1			1				

X_1	X_2	W1 old	W2 old	d=Odesired	y = Oactual	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1	0	1
1	0	0	1	1				
1	1			1				

Input 3

When (X_1=1, X_2=0)

X_1	X_2	W1 old	W2 old	d=$O_{desired}$	y = O_{actual}	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1	0	1
1	0	0	1	1				
1	1			1				

$u = \sum W_{ij} * X_j$ **(weighted sum)**

$O = f(\sum W_{ij} * X_j)$

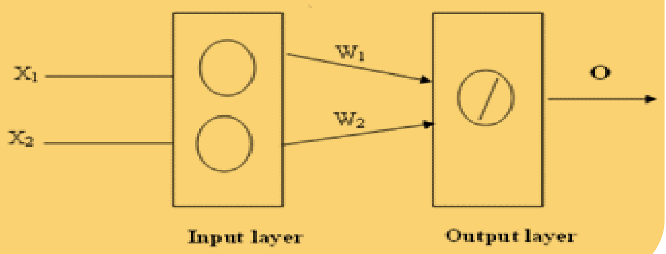
$= f(W_1 * X_1 + W_2 * X_2)$

$= f(0 * 1 + 1 * 0)$

$= f(0) \qquad u <= \theta, \quad (0 <= 0)$

$= 0$

$$y_j = f(net_j) = \begin{cases} 1 & \text{if } net_j > \theta \\ 0 & \text{if } net_j \leq \theta \end{cases} \quad \text{where} \quad net_j = \sum_{i=1}^{n} x_i w_{ij}$$



Error signal= $\triangle i$ = **E = ($O_{desired}$ - O_{actual})**

E = (d - y)

E= (1-0) = 1

X_1	X_2	W1 old	W2 old	d=$O_{desired}$	y = O_{actual}	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1	0	1
1	0	0	1	1	0	1		
1	1			1				

Input 3

When (X_1=1, X_2=0)

X_1	X_2	W1 old	W2 old	d=Odesired	y = Oactual	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1	0	1
1	0	0	1	1	0	1		
1	1			1				

X_1	X_2	W1 old	W2 old	d=Odesired	y = Oactual	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1	0	1
1	0	0	1	1	0	1	1	1
1	1			1				

X_1	X_2	W1 old	W2 old	d=Odesired	y = Oactual	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1	0	1
1	0	0	1	1	0	1	1	1
1	1	1	1	1				

Adjust the weights = update of weights

$W_{ij\ new} = W_{ij\ old} + \triangle W_{ij}$

$\triangle W_{ij} = \eta\ \triangle_i\ X_j$

$W_{ij\ new} = W_{ij\ old} + \eta\ \triangle_i\ X_j$

$W_{ij\ new} = W_{ij\ old} + \eta\ E\ X_j$

$W_{1\ new} = W_{1old} + \eta\ E\ X_1$

$W_{1\ new} = 0 + 1 * 1 * 1$

$W_{1\ new} = 1$

$W_{2\ new} = W_{2old} + \eta\ E\ X_2$

$W_{2\ new} = 1 + 1 * 1 * 0$

$W_{2new} = 1$

Input 4

When ($X_1=1, X_2=1$)

X_1	X_2	W1 old	W2 old	d=$O_{desired}$	y = O_{actual}	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1	0	1
1	0	0	1	1	0	1	1	1
1	1	1	1	1				

$u = \sum W_{ij} * X_j$ (weighted sum)

$O = f(\sum W_{ij} * X_j)$

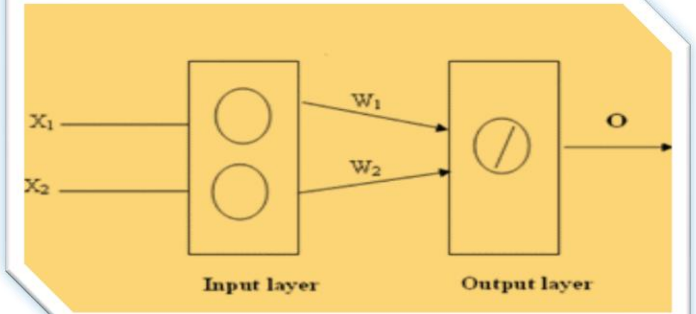
$= f(W_1 * X_1 + W_2 * X_2)$

$= f(1 * 1 + 1 * 1)$

$= f(2)$ $u > \theta$, $(2 > 0)$

$= 1$

$$y_j = f(net_j) = \begin{cases} 1 & \text{if } net_j > \theta \\ 0 & \text{if } net_j \leq \theta \end{cases} \quad \text{where} \quad net_j = \sum_{i=1}^{n} x_i w_{ij}$$



Error signal= $\triangle i$ = **E = ($O_{desired}$ - O_{actual})**

E = (d - y)

E= (1-1) = 0

X_1	X_2	W1 old	W2 old	d=$O_{desired}$	y = O_{actual}	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1	0	1
1	0	0	1	1	0	1	1	1
1	1	1	1	1	1	0		

Input 4

When (X_1=1, X_2=1)

X_1	X_2	W1 old	W2 old	d=Odesired	y = Oactual	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1	0	1
1	0	0	1	1	0	1	1	1
1	1	1	1	1	1	0		

Adjust the weights = update of weights

W_{ij} new = W_{ij} old + $\triangle W_{ij}$

$\triangle W_{ij} = \eta \triangle i X_j$

W_{ij} new = W_{ij} old + $\eta \triangle i X_j$

W_{ij} new = W_{ij} old + $\eta E X_j$

W_1 new = W_1old + $\eta E X_1$

W_1 new = 1 + 1* 0 * 1

W_1 new = 1

W_2 new = W_2old + $\eta E X_2$

W_2 new = 1 + 1* 0 * 1

W_2new = 1

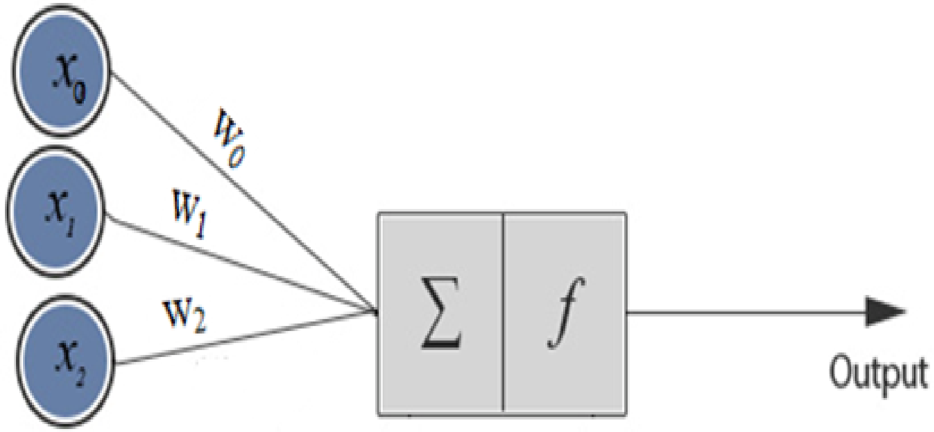
X_1	X_2	W1 old	W2 old	d=Odesired	y = Oactual	E	W1 new	W2 new
0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1	0	1
1	0	0	1	1	0	1	1	1

X_1	X_2	W1 old	W2 old	d=Odesired	y = Oactual	E	W1 new	W2 new
1	1	1	1	1	1	0	1	1

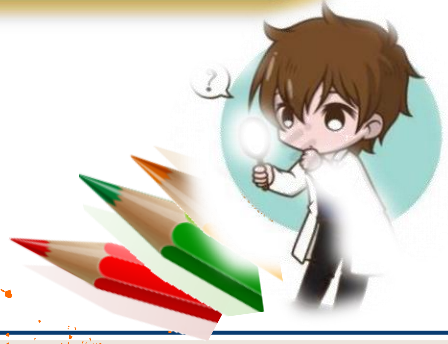
Homework

Train a perceptron network to simulate logic operation "NAND".

Learning rate η= 0.1, [w_0= 0, w_1= 0, w_2= 0], threshold (θ) = 0.5.



Inputs			AND Gate	Goal outputs
X_0	X_1	X_2	O	NOT (O= $O_{desired}$)
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0



Thank You

Any Question?

Dear students.

Please, contact via Google Classroom

Prepared by Iman Hussein