

Number Systems Operation:-

1- Decimal Numbers.

2- Binary Numbers.

3- Octal Numbers.

4- Hexadecimal Numbers.

1. **Decimal Numbers:** - In the decimal number system each of the ten digits (10digits), 0 through 9 (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9).

Decimal weight $10^4 \ 10^3 \ 10^2 \ 10^1 \ 10^0 \ 10^{-1} \ 10^{-2} \ 10^{-3}$

Example (1): $(345)_{10}$

$$300+40+5=10^2*3+10^1*4+10^0*5=345=(345)_{10}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 3 & 4 & 5 \end{array}$$

Example (2): $(234)_{10}$

$$200+30+4=10^2*2+10^1*3+10^0*4=234=(234)_{10}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 2 & 3 & 4 \end{array}$$

Example (3): $23.5 = (23.5)_{10}$

$$10^1*2 + 10^0*3 + 10^{-1}*5 = 20+3+0.5=23.5$$

Where $10^0=1$

Example (4): $45.5 = (45.5)_{10}$

$$10^1*4 + 10^0*5 + 10^{-1}*5 = 40+4+0.5=45.5$$

2- **Binary Numbers:** The binary number system its two digits a base-two system. The two binary digits (bits) are 1 and 0 (1, 0).

Binary weight $2^3 \ 2^2 \ 2^1 \ 2^0$

Weight value ... 8 4 2 1

where $2^0=1$

EX:

$(1101101)_2$

A- Binary – to – Decimal Conversion:

Example:

Convert Binary number $(1101101)_2$ into Decimal

$$(1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1)$$

$$2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 = 2^6*1 + 2^5*1 + 2^4*0 + 2^3*1 + 2^2*1 + 2^1*0 + 2^0 *1$$

$$= 64*1 + 32*1 + 16*0 + 8*1 + 4*1 + 2*0 + 1*1$$

$$= 64 + 32 + 0 + 8 + 4 + 0 + 1 \quad \Rightarrow \quad (109)_{10}$$

$$(1101101)_2 \quad \Rightarrow \quad (109)_{10}$$

Example (6):

Convert $(1001)_2$ into Decimal number

$(1 \ 0 \ 0 \ 1)_2$ Into $(\quad)_10$

$$\begin{aligned} 2^3 & 2^2 & 2^1 & 2^0 = 2^3 * 1 + 2^2 * 0 + 2^1 * 0 + 2^0 * 1 \\ & = 8 * 1 + 4 * 0 + 2 * 0 + 1 * 1 \\ & = 8 + 0 + 0 + 1 \rightarrow (9)_10 \end{aligned}$$

ملاحظة : توضع الاوزان (weight) من اليمين الى اليسار

*The Fractional binary number $(0.1011)_2$

ملاحظة : توضع الاوزان في حالة الكسر من اليسار الى اليمين

Example (7) :

Convert $(1.1011)_2$ to Decimal number

1 . 1 0 1 1

$$\begin{aligned} 2 \cdot 2^{-1} & 2^{-2} & 2^{-3} & 2^{-4} = 1 * 2 \cdot 1 * 2^{-1} + 0 * 2^{-2} + 1 * 2^{-3} + 1 * 2^{-4} \\ & = 1 \cdot 0.5 + 0 + 0.125 + 0.0625 \rightarrow (1.6875)_10 \end{aligned}$$

Example (8):

Convert $(0.1100)_2$ to Decimal number

(0 . 1 1 0 0)

$$\begin{aligned} 2 \cdot 2^{-1} & 2^{-2} & 2^{-3} & 2^{-4} = 0 * 2 \cdot 1 * 2^{-1} + 1 * 2^{-2} + 0 * 2^{-3} + 0 * 2^{-4} \\ & = 0 \cdot 0.5 + 0.125 + 0 + 0 \rightarrow (0.625)_10 \end{aligned}$$

B- Decimal – to – Binary Conversion:

- 1- Convert a decimal whole number to binary using the repeated division – by – 2
- 2- Convert a decimal fraction to binary using the repeated Multiplication – by – 2 .

Example:

Convert the number $(58)_{10}$ to Binary $(\quad)_2$

58	2	mod	LSB
29	2	→ 0	
14	2	→ 1	
7	2	→ 0	
3	2	→ 1	
1	2	→ 1	
		→ 1	MSB

(111010)₂

ملاحظة : يكتب الرقم النهائي من الاسفل الى الاعلى ومن اليسار الى اليمين
عندما يكون الرقم الناتج من القسمة اصغر من رقم الاساس نتوقف

Example:

Convert the number $(0.3125)_{10}$ to Binary ($\quad \quad \quad)_2$

	result		
$0.3125 * 2$	<u>0.6250</u>	carry	MSB
$0.6250 * 2$	<u>1.2500</u>	→ 0	
$0.2500 * 2$	<u>0.5000</u>	→ 1	
$0.5000 * 2$	<u>1.000</u>	→ 0	
0.0000		→ 1	

↓
LSB

$\longrightarrow (0101)_2$

ملاحظة : عدما نصل الى الصفر اي يكون الرفع بعد النقطة العشرية كله صفر نتوقف اما في حالة تكرار عملية الضرب لمرات كثيرة الا ان نصل الى الصفر ففي هذه الحالة نحدد عدد الارقام بعد الفاصلة العشرية مرات خمسة

مراتب بعد الفاصلة اي نقوم بعملية الضرب لخمسة مرات ونتوقف

ملاحظة : في حالة الرقم العشري يكتب الرقم النهائي من الاعلى الى الاسفل ومن اليسار الى اليمين

Example:

Convert $(20 . 625)_{10}$ to $(\quad \quad \quad)_2$

في حالة ان الرقم يحتوي على جزء صحيح وجزء كسري نقوم وبالتالي
١- نأخذ الجزء الصحيح ونجري عليه عملية القسمة على ٢ كالتالي

20	2 mod	LSB	
10	2 → 0		
5	2 → 0		
2	2 → 1		
1	2 → 0		
		↑	(10100) ₂
		MSB	

٢- ثم نأخذ الجزء الكسري ونجري عليه عملية الضرب

	result	carry	
$0.625 * 2$	1.25	1	MSB
$0.25 * 2$	0.5	0	
$0.5 * 2$	1.0	1	
0.0			↓

$\longrightarrow (.101)_2$

LSB

ثم نقوم بدمج الرقمين الثنائيين

$(10100.101)_{(2)} \rightarrow (20.625)_{10}$

3- Octal Numbers: The octal number system is composed of eight digits, which are 0, 1, 2, 3, 4, 5, 6, and 7.

A- Octal – to – Decimal conversion:

Weight $8^3 8^2 8^1 8^0$

Ex:

$(6 \ 7 \ 5)_8$

Example (11):

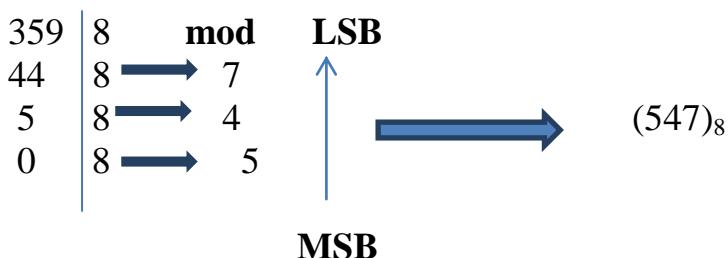
Convert Octal number $(2374)_8$ to Decimal number $(\quad)_{10}$

$$\begin{aligned}
 (2374)_8 &= 8^3 * 2 + 8^2 * 3 + 8^1 * 7 + 8^0 * 4 \\
 &= 512 * 2 + 46 * 3 + 8 * 7 + 1 * 4 \\
 &= 1024 + 192 + 56 + 4 \\
 &= (1276)_{10}
 \end{aligned}$$

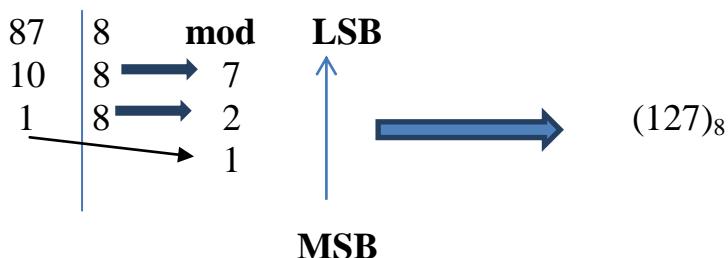
B- Decimal – to – Octal Conversion:

Example (12):

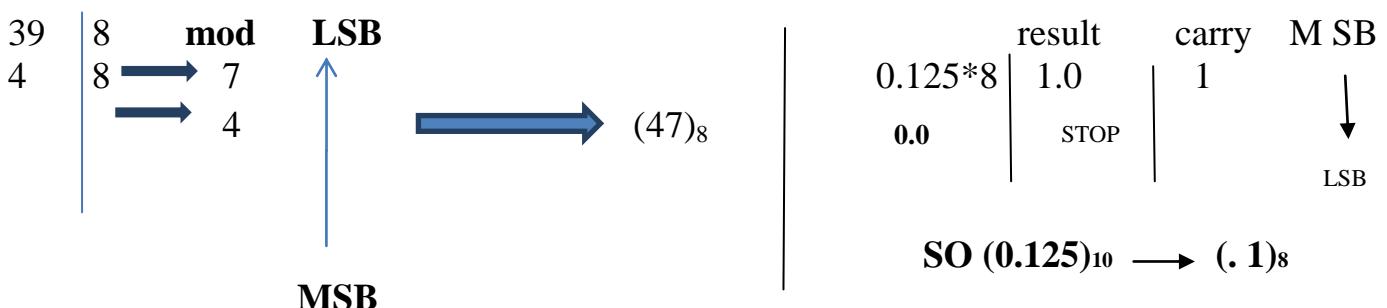
Convert decimal number $(359)_{10}$ to Octal number $(\quad)_8$



Convert decimal number $(87)_{10}$ to Octal number $(\quad)_8$



Ex: Convert decimal number $(39.125)_{10}$ to Octal number $(\quad)_8$



$(47.1)_8$ ندمج الجزئين

$$(93.125)_{10} \longrightarrow (47.1)_8$$

C- Octal – to – Binary Conversion:

Octal digit can be represented by a 3-bit binary number.

Octal digit binary

0	1	2	3	4	5	6	7
↓	↓	↓	↓	↓	↓	↓	↓

000	001	010	011	100	101	110	111
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Example

Convert $(25)_8$ to $(\quad)_2$

$$\begin{array}{l} (25)_8 \\ \downarrow \quad \downarrow \\ (010 \quad 101)_2 \end{array}$$

Example

Convert $(140)_8$ to $(\quad)_2$

$$\begin{array}{l} (1 \quad 4 \quad 0)_8 \\ (001 \ 100 \ 000)_2 \end{array}$$

example:

convert $(43.6)_8$ into $(\quad)_2$

$$\begin{array}{l} 100 \quad 011 \quad 110 \\ \swarrow \quad \downarrow \quad \searrow \\ (43.6)_8 \longrightarrow (100011.110)_2 \end{array}$$

$$(43.6)_8 \longrightarrow (100011.110)_2$$

D- Binary – to – Octal Conversion:

Conversion binary number to octal number is start with right – most groups of three bits and moving from right to left.

Example

Convert $(110101)_2$ to $(\quad)_8$

$$\begin{array}{l} 110 \quad 101 \quad 2 \\ \downarrow \quad \downarrow \quad 6 \quad 5 \quad \rightarrow (65)_8 \end{array}$$

Examples

Convert $(101111001)_2$ to $(\quad)_8$

$$\begin{array}{l} 101 \quad 111 \quad 001 \quad 2 \\ \downarrow \quad \downarrow \quad \downarrow \quad 5 \quad 7 \quad 1 \quad \rightarrow (571)_8 \end{array}$$

example

Convert $(11011.1101)_2$ to $(\quad)_8$

$$(011 \ 011 . \ 110 \ 100)$$

$$(3 \ 3 . \ 6 \ 4) \rightarrow (33.64)_8$$

4- Hexadecimal Numbers: The hexadecimal number system has a base of sixteen; it is composed of 16 digits and alphabetic characters.

Decimal	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

A- Binary – to – Hexadecimal conversion:

4-bit groups, starting at the right-most bit.

Example

Convert $(1100101001010111)_2$ to $(\quad)_{16}$

$$\begin{array}{cccc}
 \underline{1} & \underline{1} & \underline{0} & \underline{0} \\
 \text{C} & \text{A} & 5 & 7 \\
 (1100101001010111)_2 & = & = & = (\text{CA57})_{16}
 \end{array}$$

Example:

Convert $(11010110111)_2$ to $(\quad)_{16}$

$$\begin{array}{ccc}
 (0110 & 1011 & 0111)_2 \\
 \downarrow & \downarrow & \downarrow \\
 (6 & B & 7) \rightarrow (6B7)_{16}
 \end{array}$$

Example:

Convert $(110110.110)_2$ to $(\quad)_{16}$

$(0011 0110 . 1100)_2$

$$\begin{array}{ccc}
 (\downarrow & \downarrow & \downarrow) \\
 (3 & 6 & . C) \longrightarrow (36C)_{16}
 \end{array}$$

B- Hexadecimal – to – Binary Conversion:

Example (18):

Convert $(10A4)_{16}$ to $(\quad)_2$

$$\begin{array}{cccc}
 1 & 0 & A & 4 \\
 0001 & 0000 & 1010 & 0100 \\
 \end{array}$$

$(10A4)_{16} = (1000010100100)_2$

Ex.

Convert $(12C.A5)_{16}$ to $(\quad)_2$

$$\begin{array}{ccccc}
 1 & 2 & C & . & A & 5 \\
 0001 & 0010 & 1100 & . & 0101 & 0101 \\
 \end{array}$$

$(12C.A5)_{16} \rightarrow (000100101100.01010101)_2$

C- Hexadecimal – to – Decimal Conversion: By two method

* First method:

Example (19):

convert $(A85)_{16}$ to $(\quad)_{10}$

1- Convert to binary number.

2- Convert from binary number to decimal number.

$$\begin{aligned}
 & A \quad 8 \quad 5 \\
 & 1010 \quad 1000 \quad 0101 = \\
 & 2^{11}*1 + 2^{10}*0 + 2^9*1 + 2^8*0 + 2^7*1 + 2^6*0 + 2^5*0 + 2^4*0 + 2^3*0 + 2^2*1 + 2^1*0 + 2^0*1 = \\
 & 2^{11} + 2^9 + 2^7 + 2^2 + 2^0 = 2048 + 512 + 128 + 4 + 1 = 2693 = (2693)_{10} \\
 & (A85)_{16} = (2693)_{10}
 \end{aligned}$$

* Second method:

Example(20):

Covert $(E5)_{16}$ to $(\quad)_{10}$

$$\begin{aligned}
 (E5)_{16} &= 16^1*E + 16^0*5 \\
 16*14 + 1*5 &= 224 + 5 = 229 = (229)_{10} \\
 (E5)_{16} &= (229)_{10}
 \end{aligned}$$

D- Decimal – to – Hexadecimal Conversion:

Example(21) : Convert the decimal number 650 to hexadecimal by repeated division by 16.

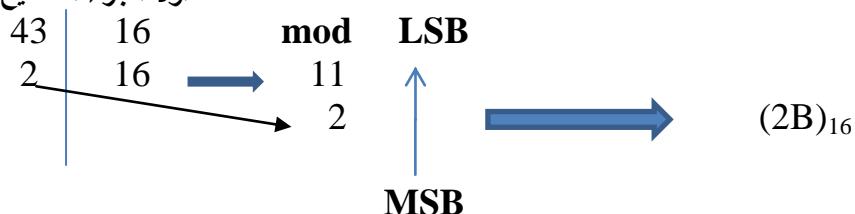
$$\begin{array}{r|rrrr}
 650 & 16 & \xrightarrow{\quad} & \text{mod} & \xrightarrow{\quad} \text{LSB} \\
 40 & 16 & \xrightarrow{\quad} & 10 & \\
 2 & 16 & \xrightarrow{\quad} & 8 & \\
 0 & 16 & \xrightarrow{\quad} & 2 & \\
 & & & & \xrightarrow{\quad} \text{MSB} \\
 \end{array}$$

$(650)_{10} = (28A)_{16}$

EX

convert $(43.125)_{10}$ into $(\quad)_{16}$

نأخذ أولاً الجزء الصحيح



ثم نأخذ الجزء الكسري

	Result	carry	MSB
$0.125 * 16$	2.0	2	
0.0	stop		LSB

ثم ندمج الجزئين $(2B.2)_{16}$

$$(43.125)_{10} \rightarrow (2B.2)_{16}$$

E- Octal to HexaDecimal

1- convert from Octal to Binary

2- convert from Binary to HexaDecimal

Ex:

Convert $(753)_8$ to $(\quad)_{16}$

$$1 - (\begin{matrix} 7 \\ 5 \\ 3 \end{matrix})_8 \\ (\begin{matrix} 111 & 101 & 011 \end{matrix})_2$$

$$2 - (0001\ 1110\ 1011)_2$$

$$(1\ E\ B)_{16}$$

$$(753)_8 \rightarrow (1EB)_{16}$$

F- HexaDecimal to Octal

1-conert from Hexa to Binary

2- convert from Binary to Octal

Ex:

(56A)₁₆ into ()₈

1- (5 6 A)₁₆

(0101 0110 1010)₂

2- (010 101 101 010)₂

(2 5 5 2)₈

(56A)₁₆ → (2552)₈

H.W

1-(1010111)₂ TO ()₁₀

2-(10100.101)₂ TO ()₁₀

3-(17)₁₀ TO ()₂

4-(155)₁₀ TO ()₂

5-(55.750)₁₀ TO ()₂

6- (205)₈ TO ()₁₀

7-(122.62)₈ TO ()₁₀

8-(87)₁₀ TO ()₈

9-(156)₁₀ TO ()₈

10-(150)₈ TO ()₂

11(235,740)₈ TO ()₂

12-(1111011)₂ to ()₈

13-(101110.11011)₂ TO ()₈

14-(110110.110)₂ TO ()₁₆

15-(11011101)₂ TO ()₁₆

16-(153.B3)₁₆ TO ()₂

17-(2A11)₁₆ TO ()₂

18-(35)₁₆ TO ()₁₀

19- (2C.43)₁₆ TO ()₁₀

17-(680)10 TO ()16

18-(72B)16 TO ()8

19-(882.2A)16 TO ()8

20-(202)8 TO ()16

21-(115. 65)8 TO ()16

المحاضرة الاولى

المرحلة الاولى

تقنيات الحاسبة

مدرسة المادة : وداد عبد الخضر