

Example: By using the bisection method find an approximation root of the equation $x^3 - x - 1.225 = 0$ that lies between $a=1$ and $b=2$. Carry out computation upto the third stage (or $\epsilon = 0.001$)

Solution:

Let $f(x) = x^3 - x - 1.225$

$f(1) = 1^3 - 1 - 1.225 = -1.225$ (-ve)

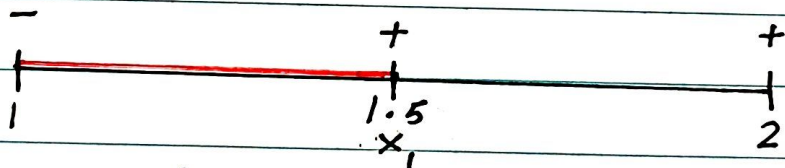
$f(2) = 2^3 - 2 - 1.225 = 4.775$ (+ve)

$\therefore$ a root lies between 1 and 2.

Thus the first approximation to the root is

$x_1 = \frac{1+2}{2} = 1.5$

and $f(x_1) = (1.5)^3 - 1.5 - 1.225 = 3.375 - 1.5 - 1.225 = 0.65$ (+ve)

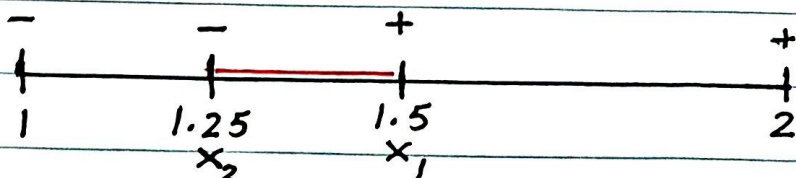


Hence the root lies between 1 and $x_1 = 1.5$.

Thus the second approximation to the root is $x_2 = \frac{1+1.5}{2} = 1.25$

and $f(x_2) = (1.25)^3 - 1.25 - 1.225$

$= 1.953125 - 1.25 - 1.225 = -0.521875$ (-ve)



Hence the root lies between $x_2 = 1.25$ and $x_1 = 1.5$.

Thus the third approximation to the root is

$$x_3 = \frac{1.25 + 1.5}{2} = 1.375$$

$$\text{and } f(x_3) = (1.375)^3 - 1.375 - 1.225$$

$$= 2.599609375 - 1.375 - 1.225 = -0.00039062501$$

$\therefore$ The root $\bar{x}$ is approximately equal 1.375, since $|f(x_3)| = 0.00039062501 < 0.001$

Example: By using the bisection method find an approximation root of the equation $e^{-x} = x$ and $\epsilon = 6\%$.

Solution: $e^{-x} = x \Rightarrow e^{-x} - x = 0$

Let $f(x) = e^{-x} - x$

$$f(0) = e^0 - 0 = 1 \quad (+ve)$$

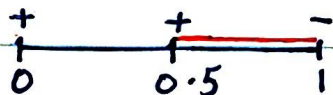
$$f(1) = e^{-1} - 1 = 0.367879441 - 1 = -0.632120558 \quad (-ve)$$

$\therefore a = 0$ and $b = 1$ and there is a root in the interval $[0, 1]$.

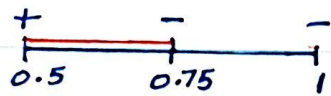
$$\therefore x_1 = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$$

$$f(x_1) = e^{-0.5} - 0.5 = 0.606530659 - 0.5 = 0.106530659 \quad (+ve)$$

$$\therefore x_2 = \frac{0.5+1}{2} = 0.75$$



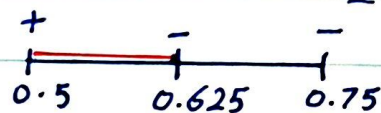
$$f(x_2) = e^{-0.75} - 0.75 = 0.472366552 - 0.75 = -0.277633447 \quad (-ve)$$



$$e\% = \left| \frac{0.75 - 0.5}{0.75} \right| * 100 \approx 33.33\%$$

$$\therefore x_3 = \frac{0.5 + 0.75}{2} = 0.625$$

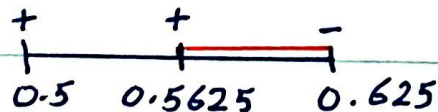
$$f(x_3) = e^{-0.625} - 0.625 = 0.535261428 - 0.625 \\ = -0.089738571 \quad (-ve)$$



$$e\% = \left| \frac{0.625 - 0.75}{0.625} \right| * 100 = 20\%$$

$$\therefore x_4 = \frac{0.5 + 0.625}{2} = 0.5625$$

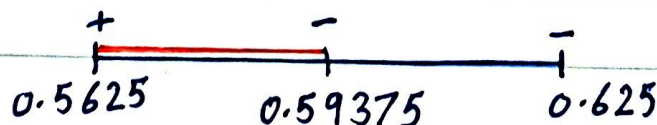
$$f(x_4) = e^{-0.5625} - 0.5625 = 0.569782824 - 0.5625 \\ = 0.007282824732 \quad (+ve)$$



$$e\% = \left| \frac{0.5625 - 0.625}{0.5625} \right| * 100 \approx 11.11\%$$

$$\therefore x_5 = \frac{0.5625 + 0.625}{2} = 0.59375$$

$$f(x_5) = e^{-0.59375} - 0.59375 \\ = 0.55225245 - 0.59375 \\ = -0.041497549 \quad (-ve)$$



$$e\% = \left| \frac{0.59375 - 0.5625}{0.59375} \right| * 100 \approx 5.26\% < \epsilon$$

$\therefore$ The root $\bar{x}$ is approximately equal 0.59375.

We can summarize the above results in the following table:

i	a	b	x_i	$e\%$	$f(a)$	$f(x_i)$	$f(a)*f(x_i)$
1	0	1	0.5	—	+	+	+
2	0.5	1	0.75	$\approx 33.33\%$	+	-	-
3	0.5	0.75	0.625	20%	+	-	-
4	0.5	0.625	0.5625	$\approx 11.11\%$	+	+	+
5	0.5625	0.625	0.59375	$\approx 5.26\%$	+	-	-

$\therefore$ The root is $\bar{x} \approx 0.59375$.

Exercises:

1) Find the approximate root of the following equation by using the bisection method:

$3x^2 = x + 5.25$ in the interval $[0.5, 3]$ and $\epsilon = 0.4$.