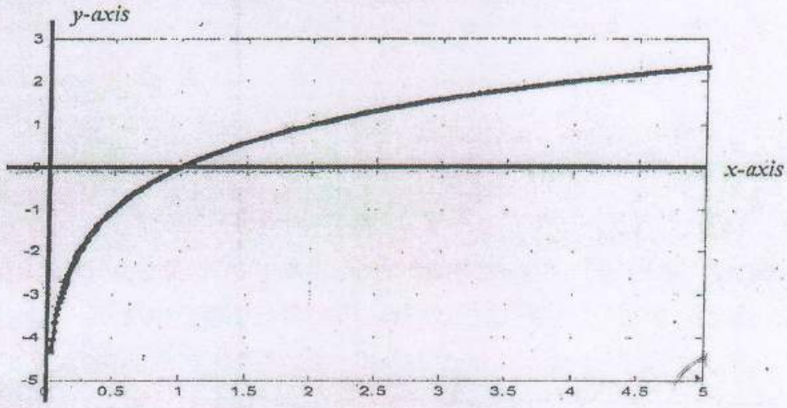
19

Remark : $y = \log_b x$ means that $x = b^y$.

Example 2.3.9 : The function $y = \log_2 x$ is a logarithm function with base 2 and its graph is

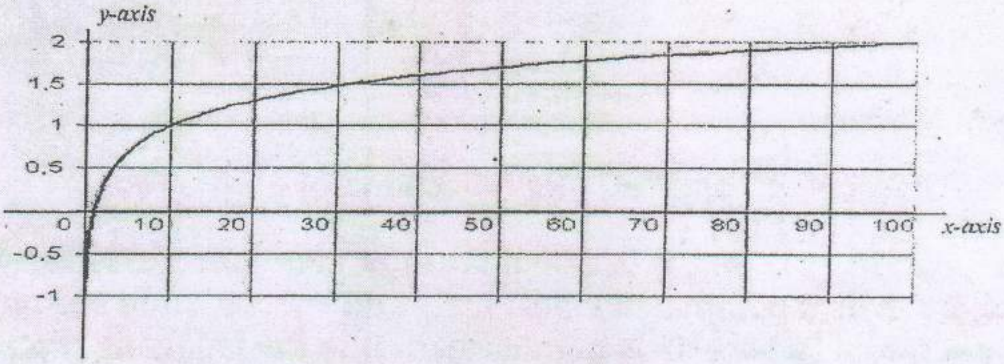
x	0.25	0.5	1	2	4
$y = \log_2 x$	−2	−1	0	1	2



Example 2.3.10 : Draw the graph of $\log_{10} x$.

Answer :

x	0.5	1	5	10	15	20	50	100
$y = \log_{10} x$	−0.301	0	0.699	1	1.176	1.301	1.699	2



Rules of logarithm : For $x > 0$ and $y > 0$, and b is a positive number $\neq 1$ we have the following rules :

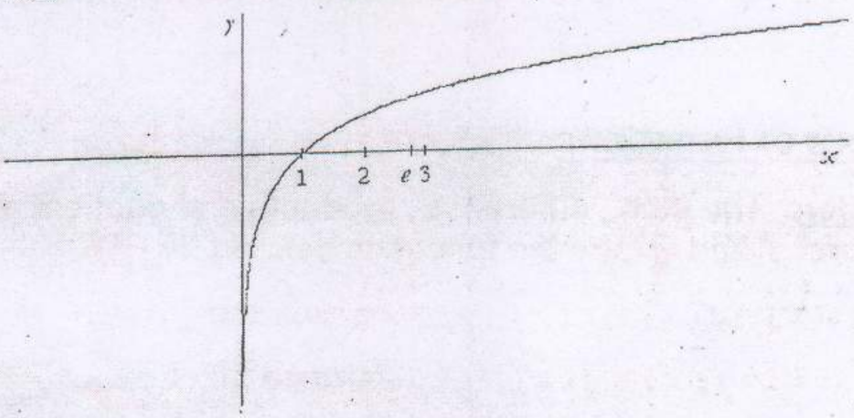
1. $\log_b xy = \log_b x + \log_b y$
2. $\log_b \frac{x}{y} = \log_b x - \log_b y$
3. $\log_b x^y = y \cdot \log_b x$
4. $\log_b a = \frac{\log_c a}{\log_c b}$, where c can be any base .

(20)

Remarks :

- The logarithm of any number to the base of the same number will be 1 ($\log_b b = 1$, $\log_5 5 = 1$ etc ...) .
- Logarithm of 1 to any base is 0 ($\log_b 1 = 0$, $\log_3 1 = 0$ etc ...) .
- The logarithm function is defined only for positive numbers .
- The domain of the logarithm function is R^+ .
- The range of the logarithm function is R .

7) The logarithm function with base e is called the natural logarithm function and will be denoted by $y = \ln x$ (i.e. $y = \log_e x = \ln x$) and its graph is



Remarks :

- $\ln e = 1$ (since $\ln e = \log_e e$)
- $\ln 1 = 0$

Exercise 2.3.12 : Draw the graph for the following logarithmic functions:

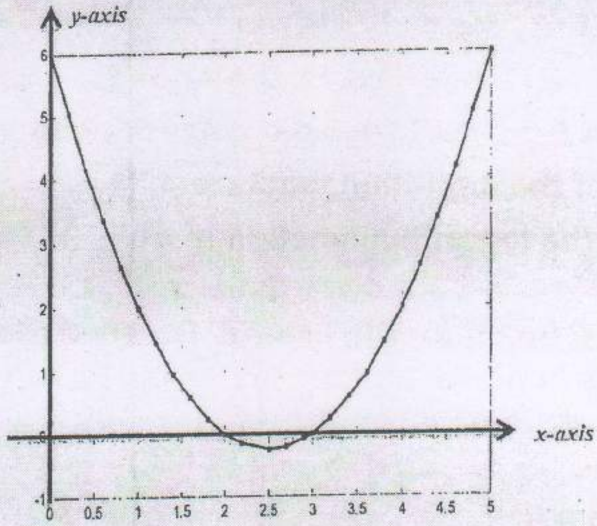
1. $\log_5 x$
2. $\log_8 x$
3. $\log_3 x$

8) A polynomial function is defined as

$y = f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ where

$a_0 , a_1 , \ldots , a_{n-1} , a_n$ are constants .

Example 2.3.13 : The function $y = x^2 - 5x + 6$ is a polynomial function .



Algebra of Functions

Definition: The sum , difference , product , and quotient of the functions f and g are the functions defined by

$(f+g)(x) = f(x) + g(x)$ sum function

$(f-g)(x) = f(x) - g(x)$ difference function

$(f.g)(x) = f(x).g(x)$ product function

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$. $g(x) \neq 0$ quotient function

The domain of each function is the intersection of the domains of f and g , with the exception that the values of x where $g(x) = 0$ must be excluded from the domain of the quotient function .

Definition: Let f and g be functions , then $f \circ g$ is called the composite of g and f and is defined by the equation $(f \circ g)(x) = f(g(x))$.

The domain of $f \circ g$ is the set
$\mathbf{D} = \{ x \in \text{domain } g : g(x) \in \text{domain } f \}$.

Example 2.3.14 : Let f and g be the functions defined by $f(x) = x - 7$ and $g(x) = x^2 + 5$. Find the functions $f+g$, $f-g$, $f.g$, $\frac{g}{f}$, $f \circ g$, $g \circ f$ and find their domains .

Solution :

$(f+g)(x)=f(x)+g(x) = x-7+x^2+5 = x^2+x-2$
$(f-g)(x)=f(x)-g(x) = x-7 - x^2-5 = -x^2+x-12$

$(f.g)(x)=f(x).g(x) = (x-7).(x^2+5) = x^3-7x^2+5x-35$

$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{x^2+5}{x-7}$

$(f \circ g)(x)=f(g(x)) = f(x^2+5) = x^2+5-7 = x^2-2$

$(g \circ f)(x)= g(f(x)) = g(x-7) = (x-7)^2+5$
$= x^2-14x+49+5 = x^2-14x+54$

The domain of $f = \mathrm{R}$

The domain of $g = \mathrm{R}$

The intersection of the domains of f and g is R

Thus the domain of each of the functions $f+g$, $f-g$, $f.g$, $f \circ g$, and $g \circ f$ is R .

The domain of $\frac{g}{f} = \mathrm{R}-\{7\}$.

Remark : The domain of any polynomial function is R .

Example 2.3.15 : Let f and g be the functions defined by $f(x)=x+5$ and $g(x)=x^2-3$, Find $f \circ g(x)$, $g \circ f(x)$, $f \circ g(3)$ and $g \circ f(3)$.

Solution: $f \circ g(x) = f(g(x)) = f(x^2-3)$
$= x^2-3+5$
$= x^2+2$

$g \circ f(x) = g(f(x)) = g(x+5)$
$= (x+5)^2-3$
$= x^2+10x+25-3$
$= x^2+10x+22$

$$f \circ g(3) = (3)^2 + 2 = 9 + 2 = 11$$

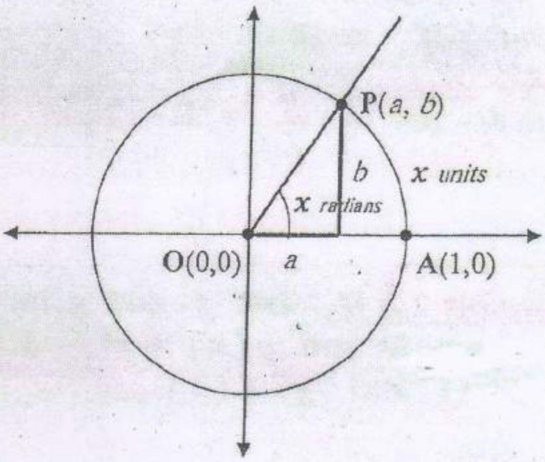
$$g \circ f(3) = (3)^2 + 10(3) + 22 = 9 + 30 + 22 = 61$$

<u>Exersice 2.3.16 :</u> Let f and g be the functions defined by $f(x) = x - 4$ and $g(x) = \sqrt{x}$. Find the functions $f + g$, $f - g$, $f \cdot g$, $\frac{f}{g}$ and find their domains .

<u>S 2.4 : Unit Circle and Basic Trigonometric Functions</u>

<u>Definition 1:</u> Let x be any real number and let U be the unit circle with equation $a^2 + b^2 = 1$ (the centre of the circle U is the point O (0,0) , and the radius of the circle U equals 1) . Start from the point A(1,0) on U and proceed counterclockwise if x is positive and clockwise if x is negative around the unit circle U until an arc length of $|x|$ has been covered . Let P(a, b) be the point at the terminal end of the arc . The measurement of the angle AOP is x radians .

If x radians = t° (degrees), then the following six trigonometric functions of x are defined in terms of the coordinates of the circular point P(a, b) :



1) $y = \sin x = b = \sin(x \text{ radians}) = \sin(t \text{ degrees}) = \sin t^\circ$

2) $y = \cos x = a = \cos(x \text{ radians}) = \cos(t \text{ degrees}) = \cos t^\circ$

3) $y = \tan x = \dfrac{b}{a} \qquad (a \neq 0)$

$= \tan(x \text{ radians}) = \tan(t \text{ degrees}) = \tan t^\circ$

4) $y = \cot x = \dfrac{a}{b} \qquad (b \neq 0)$

$= \cot(x \text{ radians}) = \cot(t \text{ degrees}) = \cot t^\circ$