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Exercise: By N-R method find the solution of the following equation, correct to $|X_{i+1} - X_i| < 0.001$ and use $X_0 = 2$?

4- Fixed point iteration method :-

- Given $f(x) = 0$, write x in terms of $x = g(x)$.
- Label left side as x_{i+1} and right side with x_i.
- Pick x_1 and plug into equation.
- Repeat until converges.

Example 1:

By the fixed point iteration method, find the value of the root of $f(x) = x^2 + 2x - 1$ correct to $|X_{i+1} - X_i| < 0.001$?

Solution

x	0	1
$f(x)$	$-$	$+$

$$X_0 = \frac{0+1}{2} = 0.5$$

(a) $x^2 + 2x - 1 = 0$

$$x^2 = 1 - 2x$$

$$x = \underbrace{\sqrt{1-2x}}_{g(x)}$$

$$g'(x) = \frac{-1}{\sqrt{1-2x}}$$

$$g'(x_0) = \frac{-1}{\sqrt{1-2*0.5}} = \frac{-1}{0} = \infty \quad (\text{divergent})$$

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(b) $2x = 1 - x^2$

$x = \dfrac{1-x^2}{2}$

$g(x) = \dfrac{1-x^2}{2}$

$g'(x) = -x$

$g'(x_0) = -0.5$

$|g'(x_0)| = 0.5 < 1$ (convergent

(c) $x^2 + 2x - 1 = 0$

$x(x+2) - 1 = 0$

$x = \dfrac{1}{x+2} \Rightarrow g(x) = \dfrac{1}{x+2}$

$g'(x) = \dfrac{-1}{(x+2)^2}$

$g'(x_0) = \dfrac{-1}{(2.5)^2} = \dfrac{-1}{6.25} = -0.16$

$\therefore |g'(x)| = 0.16 < 1$ (Convergent)

<u>Remark</u>: Sometimes we can write f(x) in many formulas of g(x), some of these g(x) convergence to the root and other of g(x) divergence from the root. The following testing defines the best formulas which convergent to the root:

$|g'(x_0)| < 1$

we select the formula:

$$g(x) = \frac{1}{x+2}.$$

$$x_{i+1} = \frac{1}{x_i + 2}, \quad x_0 = 0.5$$

i	x_i	x_{i+1}	$\lvert x_{i+1} - x_i \rvert$
0	0.5	0.4	—
1	0.4	0.410	0.010
2	0.410	0.414	0.004
3	0.414	0.414	0

$\therefore$ The root $\bar{x} \simeq 0.414$

Exercise:

Find the root of the equation $\sin x - x^2 + 1 = 0$ by fixed point iteration method correct to $\lvert x_{i+1} - x_i \rvert < 0.002$ and $x_0 = 1.5$?

5- Secant method :-

This method is similar to the false position method, a general formula of this method is defined by:

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Example 1: By the Secant method, find the approximate value of the root for the following equation: $x\ln x - 1 = 0$ in the interval $[2,3]$, corret to $|f(x_{i+1})| < 0.001$?

Solution :-

$f(x) = x\ln x - 1$, $[2,3]$, $|f(x_{i+1})| < 0.001$

$x_0 = 2 \Rightarrow f(x_0) = 0.386$

$x_1 = 3 \Rightarrow f(x_1) = 2.296$

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)} = 1.798 \Rightarrow f(x_2) = 0.055$$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)} = 1.769 \Rightarrow f(x_3) = 0.004$$

$$X_4 = X_3 - \frac{f(X_3)(X_3 - X_2)}{f(X_3) - f(X_2)} = 1.767$$

$$\Rightarrow f(X_4) = 0.0003$$

$$\therefore |f(X_4)| = |0.0003| < 0.001$$

$\therefore$ The root $\bar{X} = 1.767$

Exercise:-

Find approximate value of the root for the following equation:

$X^3 - 20 = 0$, $X_0 = 4$, $X_1 = 5.5$ Correct to two loops?