2. The false position method

This method is similar to the bisection method.
It requires two initial values a and b.

Algorithm steps :-

1. choose an interval $[a,b]$ such that
$$f(a) * f(b) < 0$$

2. Find (x_i) as an instantaneous root:-
$$x_i = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

3. Find and calculate $f(x_i)$ by using x_i-value

4. If $f(a) * f(x_i) < 0 \Rightarrow b = x_i$ and $f(b) = f(x_i)$
If $f(a) * f(x_i) > 0 \Rightarrow a = x_i$ and $f(a) = f(x_i)$

5. Repeat the above procedure starting from step (2) to calculate a new (x_i) ... and so on.

6. Terminate the calculations when the given accuracy condition is satisfied.

Example Find approximate value of roots by using false position method of the following equation $f(x) = e^x - 3x$ in the interval $[1,2]$ and $\epsilon = 0.01$?

Solution :-

We have $a = 1$ and $b = 2$

$f(a) = f(1) = e^1 - 3(1) = 2.71 - 3 = -0.88$

$f(b) = f(2) = e^2 - 3(2) = 1.389$

$\therefore f(a) * f(b) < 0$

$\therefore$ there is a root in the interval $[1,2]$.

We have: $$x_i = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

i	a	b	$f(a)$	$f(b)$	x_i	$f(x_i)$	$f(a) * f(x_i)$
1	1	2	-0.281	1.389	1.169	-0.288	+
2	1.169	2	-0.288	1.389	1.311	-0.223	+
3	1.311	2	-0.223	1.389	1.406	-0.138	+
4	1.406	2	-0.138	1.389	1.459	-0.075	+
5	1.459	2	-0.075	1.389	1.486	-0.038	+
6	1.486	2	-0.038	1.389	1.499	-0.019	+
7	1.499	2	-0.019	1.389	1.505	-0.0108	+
8	1.505	2	-0.0108	1.389	1.502	-0.004	

$\therefore$ the root $\vec{x} \simeq 1.509$

Exercise:- (16)

Solve the following equation using false position method:

$f(x) = x^3 - 2x^2 - x - 1$ in the interval $[-1, 0]$ and $\epsilon = 0.001$?

3- Newton- Raphson method:-

Let $f(x)$ be a continuous and differentialbe function, x_0 the intial approximate value of the root to the equation. therefore a new root can be defined as follows:

$$X_1 = X_0 + h$$

where h is the correction value of the root:

$$f(x_1) = f(x_0 + h)$$

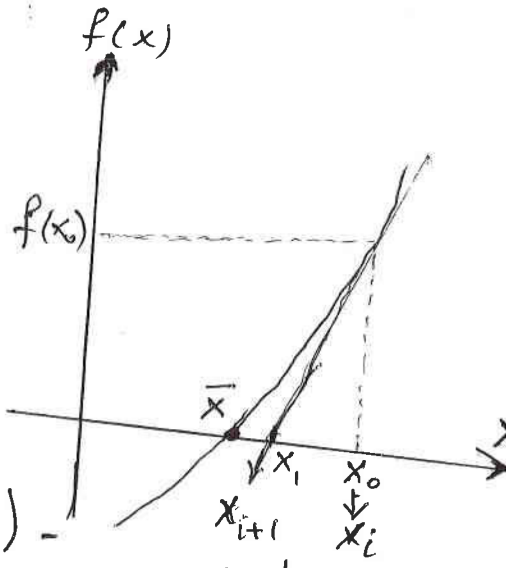
by using Taylor's formula at the point X_0, we get:

$$f(x_1) = f(x_0) + (X_1 - X_0)\frac{f'(x_0)}{1!} + (X_1 - X_0)^2 \frac{f''(x_0)}{2!} + \cdots$$

by iteration this proceedure, we will get a general formula of the Newton-Raphson method:

$$X_{i+1} = X_i - \frac{f(x_i)}{f'(x_i)} \quad , i = 0, 1, 2, \cdots$$

(17)



The algorithm of the method:-

1- Take an initial value (x_0).
2. Calculate $f(x_0)$ and $f'(x_0)$
3. Calculate the intersection (x_1).
4. Put $x_0 = x_1$ and Calculate the new intersection x_2 by the same procedure.
5. Repeat the process to get $x_3, x_4, \ldots$ until reaching the required accurracy.

* in general $\quad x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)} \quad , i = 0, 1, 2, \ldots$

Example 1. :

Find the root of $f(x) = e^x - 3x$ in the interval $[0,1]$. Correct to $|x_{i+1} - x_i| < 0.005$, use Newton-Raphson method with $x_0 = 0$.

Solution :-

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x_i) = e^{x_i} - 3x_i \quad , \quad f'(x) = e^x - 3$$

$$x_{i+1} = x_i - \frac{e^{x_i} - 3x_i}{e^{x_i} - 3} \quad , i = 0, 1, 2, \ldots$$

i	x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}	$\lvert x_{i+1}-x_i \rvert$
0	0	1	-2	0.5	—
1	0.5	0.1487	-1.3512	0.61	0.11
2	0.61	0.0104	-1.1595	0.6156	0.0056
3	0.6156	$3.97 * 10^{-3}$	-1.1492	0.619	0.0034

∴ The root $\bar{x} \simeq 0.619$.

Example 2:- Find the root of the equation: $(x-2)^2 = x + 54$ by N-R method correct to two loops, use $x_0 = 8$?

Solution

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, \quad i = 0, 1, 2, \ldots$$

$$f(x_i) = x_i^2 - 5x_i - 50$$

$$f'(x_i) = 2x_i - 5$$

i	x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}
0	8	-26	11	10.3636
1	10.3636	5.5862	15.7272	10.0084
2	10.0084	0.1260	15.0168	10.0000

∴ $\bar{x} = 10.0000$