PART ONE

STATISTICS AND PROBABILITY

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Chapter One

Introduction to probability

Some important definitions:

Probability theory:

It is a type of mathematics dealing with random experiments.

Random experiment:

Is the experiment whose respected observations under a specified set of conditions does not always leads to the same outcome.

Sample space:

The set of all possible outcomes of an experiment, it is denoted by "S".

Event:

It is indicates an outcome or collection of outcomes in any random experiment, or it is a subset of the sample space.
For example:
If an event, "*C*" can happen in "*f*" ways out of a total "*N*" possible equally likely ways, then the probability of occurrence of the event called its *success* and denoted by:

$$P(C) = \frac{f}{N}$$

and the probability of non–occurrence of the event called it's *failure* and denoted q such that: q=P(not C), q=1–P(C)
where p + q=1

Examples:

1. Two dice thrown once let *A* be the event of collection of every pair of the sample space for which the sum of the pair is equal to *seven*.

Solution: The sample space is:

S={(1,1),(1,2),…,(1,6),(2,1),…(2,6),…,(6,1),…,(6,6)}=36, and
A={(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)}=6
Then $P(A) = \frac{6}{36} = \frac{1}{6}$

2. A random experiment of throwing a coin three times. Write the sample space and the subset of the event that the coin turns up the same for all three times.

Solution:
Let T denote "Tail", and H denote "Head" so the sample space is:
S= {TTT, HHH, TTH, THT, HTT,THH, HTH, HHT}=8
A= {TTT, HHH} =2

Classification the types of events:

1–Simple event: For example, Tail or Head appears when a coin is thrown.
2–Mutually exclusive events: if A and B are two mutually exclusive events then: P(AB)=0

for example a coin is thrown then a Tail is appears that not a Head appears.

3–Dependent and independent events: if A and B are two independent events then A appearing not affect that B appearing then: $P(AB)=P(A)*P(B)$

Relations between events:

1. The union of two events A and B is the event contains of all the elements that either in A or in B or in both events, denoted by:
   $(A \cup B)$, (A or B), or $(A+B)$.
2. The intersection of two events A and B is the event contains of all the elements that are in both A and B, denoted by: $(A \cap B)$, (A and B), or $(A*B)$.
3. The complement of an event A is the set of all elements in sample space that are not in A, denoted by A^c.

Example:

A fair die thrown once, suppose that:
A_1: even numbers.
A_2: numbers greater than 3.
A_3: Odd numbers.
Find: $A_1 \cap A_2$, $A_1 \cap A_3$, $A_1 \cup A_2$, A_1^c .
Solution:
$S= \{1, 2, 3, 4, 5, 6\}$
$A_1= \{2, 4, 6\}$
$A_2= \{4, 5, 6\}$
$A_3 = \{1, 3, 5\}$
$A_1 \cap A_2 = \{4, 6\}$
$A_1 \cap A_3 = \{\phi\}$
$A_1 \cup A_2 = \{2, 4, 5, 6\}$
$A_1^c = \{1, 3, 5\} = A_3$

Some laws of events:

For A, B, and C are three events:

1. $(A \cup B)=(B \cup A)$ and $(A \cap B)=(B \cap A)$
2. $A\cup(B\cup C)=(A\cup B) \cup C$ and $A\cap (B\cap C)=(A\cap B) \cap C$
3. $A \cap (B\cup C)=(A\cap B) \cup(A\cap C)$ and
   $A\cup (B\cap C) = (A\cup B) \cap(A\cup C)$
4. $A \cup \phi =A$ and $A \cap \phi = \phi$
   $A \cup S = S$ and $A \cap S = A$
5. $A \cup A^c = S$ and $A \cap A^c = \phi$
   $S^c = \phi$ and $\phi^c = S$ and $(A^c)^c = A$

$(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c$