## <u>Exercise(1): -</u> Given Example

1- Quotient ring is a field
2- Quotient ring is an integral domain
3- Quotient ring has no zero divisors
4- Quotient ring is a comm..ring with identity but is not field

**<u>Solution(1):-</u>** $(Z_{28}/(\bar{7})\,,+,.)$ is field

Since,( $(\bar{7})$,$+_7$,$\cdot_7$) is a maximal ideal in a ring $(Z_{28}, +_{28}, \cdot_{28})$,then by Th.(2-15) we get $(Z_{28}/(\bar{7})\,,+,.)$ is field .

**<u>Solution(2):-</u>** $(Z_{28}/(\bar{7})\,,+,.)$ is an integral domain , since $(Z_{28}/(\bar{7})\,,+,.)$ is field field [ Since,( $(\bar{7})$,$+_7$,$\cdot_7$) is a maximal ideal in a ring $(Z_{28}, +_{28}, \cdot_{28})$,then by Th.(2-15) we get $(Z_{28}/(\bar{7})\,,+,.)$ is field ] and by Th(1-4) [ every field is an integral domain ] , so we get$(Z_{28}/(\bar{7})\,,+,.)$ is an integral domain .

**<u>Solution(3):-</u>**

$(Z_{28}/(\bar{7})\,,+,.)$ is a quotient ring has no zero divisor , since

$(Z_{28}/(\bar{7})\,,+,.)$ is an integral domain , because $(Z_{28}/(\bar{7})\,,+,.)$ is field field [ Since,( $(\bar{7})$,$+_7$,$\cdot_7$) is a maximal ideal in a ring $(Z_{28}, +_{28}, \cdot_{28})$,then by Th.(2-15) we get $(Z_{28}/(\bar{7})\,,+,.)$ is field ] and by Th(1-4) [ every field is an integral domain ] , so we get$(Z_{28}/(\bar{7})\,,+,.)$ is an integral domain then by definition of integral domain we get $(Z_{28}/(\bar{7})\,,+,.)$ is a quotient ring has no zero divisor .

**<u>Solution (4):-</u>**

$(Z_{20}/(\overline{10})\,,+,.)$ is a comm..ring with identity , but is not integral

Domain ,since $(Z_{20}/(\overline{10})\,,+,.)$ has zero divisors