Corollary :-

Every order Field contains the Field of rational numbers.

proof :- (Exe).

Example :- Show that the equation $x^2 = 2$ has no roots in the Field rational numbers.

proof :- Suppose that y is rational number $\ni y^2 = 2$

Let $y = \frac{m}{n}$, m, n are positive integer and $n \neq 0$

$g.c.d(m,n) = 1$.

$$\left(\frac{m}{n}\right)^2 = 2 \Rightarrow \frac{m^2}{n^2} = 2 \Rightarrow m^2 = 2n^2.$$

1- Suppose m is even and n is odd.

$m = 2K \Rightarrow m^2 = 4K^2$.

$4K^2 = 2n^2 \Rightarrow 2K^2 = n^2 \quad (\div 2)$.

We get n^2 is even.

but n is odd $\Rightarrow n^2$ is odd

2. suppose m is odd and n is even.

$n = 2K \Rightarrow n^2 = 4K^2$

$\therefore\ m^2 = 2 \cdot 4K^2 \Rightarrow m^2$ is even.

but m is odd $\Rightarrow m^2$ is odd.

$\therefore\ m^2$ is even and odd which is contradiction (C!)

3. suppose m and n are odd.

$m^2 = 2n^2 \Rightarrow m^2$ is even.

But m is odd $\Rightarrow m^2$ is odd.

$\therefore\ m^2$ is even and odd which is contradition (C!).

There is no radional number which is satisfy the equation $x^2 = 2$. ($\nexists$ a solution of the equation $x^2 = 2$ in $\mathbb{Q}$).