Definition :

let $S$ be an order Set, $E \subseteq S$ such that $E$ is bounded above.

An element $\gamma \in S$ is called least upper bound (L.u.b.) of $E$ if :-

1. $x \leq \gamma \quad \forall x \in E$.
2. $\gamma \leq y \quad \forall$ upper bound $y$ of $E$.

Ex:- ① let $S = \mathbb{R}$, $E = \{x : 1 \leq x \leq 4\}$

each element of $\{x : x \geq 4\}$ is upper bound of $E$.

4 is least upper bound of $E$.

We have $L.u.b.(E) = 4$.

**Definition** : Let $S$ be an order Set, $E \subseteq S$ such that $E$ is bounded below.

An element $\beta \in S$ is called greatest Lower bound (g.L.b.) of $E$ if :-

1. $\beta \leq x \quad \forall x \in E$.
2. $\beta \geq y \quad \forall$ Lower bound of $E$.

Ex: ① $S = \mathbb{R}$, $E = \{x : -2 \leq x \leq 7\}$

An element of set $\{x : x \leq -2\}$ is Lower bound of $E$

$-2$ is greatest Lower bound of $E$.

$g.L.b.(E) = -2$

② $S = \mathbb{Q}$, $E = \{1, 2, 3, \ldots\}$.

The set of Lower bound of $E$ is $\{x : x \leq 1\}$

$1$ is greatest Lower bound of $E$.

$\therefore \; g.L.b(E) = 1$.

③ $S = \mathbb{R}$, $E = \{\ldots, -3, -2, -1, 0, 1\}$.

since $E$ is not bounded below

Therefore $E$ no has $g.L.b$.