

## FIRST and FOLLOW

The construction of a predictive parser is aided by two functions associated with a grammar  $G$ . These functions, **FIRST** and **FOLLOW**, allow us to fill in the entries of a predictive parsing table for  $G$ , whenever possible.

### First Function

To compute **FIRST**( $X$ ) for all grammar symbols  $X$ , apply the following rules until no more terminals or  $\epsilon$  can be added to any **FIRST** set.

- 1) If  $X$  is **Terminal**, then **FIRST**( $X$ ) is  $\{X\}$ .
- 2) If  $X \longrightarrow \epsilon$  is a production, then add  $\epsilon$  to **FIRST**( $X$ ).
- 3) If  $X$  is **Nonterminal** and  $X \longrightarrow Y_1 Y_2 \dots Y_k$  is a production, then place  $\underline{a}$  in **FIRST**( $X$ ) if for some  $i$ ,  $\underline{a}$  is in **FIRST**( $Y_i$ ), and  $\epsilon$  is in all of **FIRST**( $Y_1$ ),  $\dots$ , **FIRST**( $Y_{i-1}$ ); that is,  $Y_1 \dots Y_{i-1} \xrightarrow{*} \epsilon$ . If  $\epsilon$  is in **FIRST**( $Y_j$ ) for all  $j = 1, 2, \dots, k$ , then add  $\epsilon$  to **FIRST**( $X$ ). For example, everything in **FIRST**( $Y_1$ ) is surely in **FIRST**( $X$ ). If  $Y_1$  does not derive  $\epsilon$ , then we add nothing more to **FIRST**( $X$ ), but if  $Y_1 \xrightarrow{*} \epsilon$ , then we add **FIRST**( $Y_2$ ) and so on.

Now, we can compute **FIRST** for any string  $X_1 X_2 \dots X_n$  as follows. Add to **FIRST**( $X_1 X_2 \dots X_n$ ) all the non- $\epsilon$  symbols of **FIRST**( $X_1$ ). Also add the non- $\epsilon$  symbols of **FIRST**( $X_2$ ) if  $\epsilon$  is in **FIRST**( $X_1$ ), the non- $\epsilon$  symbols of **FIRST**( $X_3$ ) if  $\epsilon$  is in both **FIRST**( $X_1$ ) and **FIRST**( $X_2$ ), and so on. Finally, add  $\epsilon$  to **FIRST**( $X_1 X_2 \dots X_n$ ) if, for all  $i$ , **FIRST**( $X_i$ ) contains  $\epsilon$ .

**First Function Examples:****Example1:** Consider the following grammar

$$S \longrightarrow aSb \mid ba \mid \mathcal{E}$$

$$\text{FIRST}(S) = \{a, b, \mathcal{E}\}$$

**Example2:** Consider the following grammar

$$S \longrightarrow TabS \mid X$$

$$T \longrightarrow cT \mid \mathcal{E}$$

$$X \longrightarrow b \mid bX$$

**Sol:**

$$\text{FIRST}(S) = \{c, a, b\}$$

$$\text{FIRST}(T) = \{c, \mathcal{E}\}$$

$$\text{FIRST}(X) = \{b\}$$

**Example3:** Consider the following grammar

$$S \longrightarrow AB \mid bS$$

$$A \longrightarrow aB \mid BB$$

$$B \longrightarrow b \mid cB$$

**Sol:**

$$\text{FIRST}(S) = \{a, b, c\}$$

$$\text{FIRST}(A) = \{a, b, c\}$$

$$\text{FIRST}(B) = \{b, c\}$$

**Example4:** Consider the following grammar

$$S \longrightarrow XYB \mid ccb$$

$$X \longrightarrow xX \mid \mathcal{E}$$

$$Y \longrightarrow yY \mid Xy \mid \mathcal{E}$$

$$B \longrightarrow bbc \mid b$$

**Sol:**

$$\text{FIRST}(S) = \{c, x, y, b\}$$

$$\text{FIRST}(X) = \{x, \mathcal{E}\}$$

$$\text{FIRST}(Y) = \{y, x, \mathcal{E}\}$$

$$\text{FIRST}(B) = \{b\}$$

**Example5:** Consider the following grammar

$$\begin{aligned} S &\longrightarrow \mathbf{abS} \mid \mathbf{bX} \\ X &\longrightarrow \mathbf{\epsilon} \mid \mathbf{cN} \\ N &\longrightarrow \mathbf{Nb} \mid \mathbf{c} \end{aligned}$$

There is the left recursion problem, so we must solve this problem before finding the first function

$$\begin{aligned} S &\longrightarrow \mathbf{abS} \mid \mathbf{bX} \\ X &\longrightarrow \mathbf{\epsilon} \mid \mathbf{cN} \\ N &\longrightarrow \mathbf{cN'} \\ N' &\longrightarrow \mathbf{bN'} \mid \mathbf{\epsilon} \end{aligned}$$

**Sol:**

$$\text{FIRST}(S) = \{\mathbf{a}, \mathbf{b}\}$$

$$\text{FIRST}(X) = \{\mathbf{c}, \mathbf{\epsilon}\}$$

$$\text{FIRST}(N) = \{\mathbf{c}\}$$

$$\text{FIRST}(N') = \{\mathbf{b}, \mathbf{\epsilon}\}$$

### Follow Function

To compute  $\text{FOLLOW}(A)$  for all nonterminals  $A$ , apply the following rules until nothing can be added to any  $\text{FOLLOW}$  set.

- 1) Place  $\$$  in  $\text{FOLLOW}(S)$ , where  $S$  is the start symbol And  $\$$  is the input right endmarker.
- 2) If there is a production  $A \longrightarrow \mathbf{aB\beta}$ , then everything in  $\text{FIRST}(\beta)$  except for  $\mathbf{\epsilon}$  is placed in  $\text{FOLLOW}(B)$ .
- 3) If there is a production  $A \longrightarrow \mathbf{aB}$ , or a production  $A \longrightarrow \mathbf{aB\beta}$  where  $\text{FIRST}(\beta)$  contains  $\mathbf{\epsilon}$  (i.e.,  $\beta \xrightarrow{*} \mathbf{\epsilon}$ ), then everything in  $\text{FOLLOW}(A)$  is in  $\text{FOLLOW}(B)$ .

**Follow Function Examples:****Example1:** Consider the following grammar

$$S \longrightarrow bXY$$

$$X \longrightarrow b \mid c$$

$$Y \longrightarrow b \mid \epsilon$$

Nonterminals	First	Follow
$S$	$b$	$\$$
$X$	$b, c$	$b, \$$
$Y$	$b, \epsilon$	$\$$

**Example2:** Consider the following grammar

$$S \longrightarrow aSb \mid X$$

$$X \longrightarrow cXb \mid b$$

$$X \longrightarrow bXZ$$

$$Z \longrightarrow n$$

Nonterminals	First	Follow
$S$	$a, b, c$	$\$, b$
$X$	$b, c$	$b, n, \$$
$Z$	$n$	$b, n, \$$

**Example3:** Consider the following grammar

$$S \longrightarrow ABb \mid bc$$

$$A \longrightarrow abAB \mid \epsilon$$

$$B \longrightarrow bc \mid cBS$$

Nonterminals	First	Follow
$S$	$a, b, c$	$\$, a, b, c$
$A$	$a, \epsilon$	$b, c$
$B$	$b, c$	$a, b, c$