Determinants

Def: Let A = [a_{ij}] be an nxn matrix , we define the determinant of A (written $|A|$ by or (det (A))

$$|A| = \sum (\pm)\, a_{1j1}\, a_{2j2} \ldots\ldots\ldots\ldots a_{njn}$$

Ex// if A =[a_{11}] is 1x1 matrix then

$$|A| = a_{11}$$

1- A = [4] $\Longrightarrow$ $|A| = 4$

2- A = [-6] $\Longrightarrow$ $|A| = -6$

Ex// $A = \begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}$ $2x2$

Det (A) = $a_{11}\, a_{22} - a_{12}\, a_{21}$

$A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$

Det (A) = (2)(5) – ((-3)(4)) = 22

Ex// $A = \begin{bmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{bmatrix} \begin{matrix} a11 & a12 \\ a21 & a22 \\ a31 & a32 \end{matrix}$

$$|A| = (a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}) - (a_{13}a_{22}a_{31} + a_{11}a_{23}a_{32} + a_{12}a_{21}a_{33})$$

Ex// $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{bmatrix} \begin{matrix} 1 & 2 \\ 2 & 1 \\ 3 & 1 \end{matrix}$

Det (A) = (2+18+6) – (8+ 3+ 9) = 26 -20 = 6

Properties of Determinant

1- The determents of matrix and its transpose are equal

Ex// Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{bmatrix} \begin{matrix} 1 & 2 \\ 2 & 1 \\ 3 & 1 \end{matrix}$ $|A| = 6$

$A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 3 & 2 \end{bmatrix} \begin{matrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \end{matrix}$ $= (2+6+18) - (8 + 3 + 9) = 6$

$$|A| = |A^T|$$