

DEF : if $A = [a_{ij}]$ is an $m \times n$ matrix and r is a real number the rA is the $m \times n$ matrix $B = [b_{ij}]$ where $b_{ij} = ra_{ij}$ ($1 \leq i \leq m, 1 \leq j \leq n$)

$$\text{Ex// if } r = -3 \text{ and } A = \begin{bmatrix} 4 & -2 & 3 \\ 2 & -5 & 0 \\ 3 & 6 & -2 \end{bmatrix}$$

$$\text{Then } rA = -3 \begin{bmatrix} 4 & -2 & 3 \\ 2 & -5 & 0 \\ 3 & 6 & -2 \end{bmatrix} = \begin{bmatrix} -12 & 6 & -9 \\ -6 & 15 & 0 \\ -9 & -18 & 6 \end{bmatrix}$$

$$A = [4 \ 2 \ 3] \quad B = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$$

$$AB = [(4)(1) + (2)(0) + (3)(5)] = [19]$$

The Transpose

If $A = [a_{ij}]$ is an $m \times n$ matrix , the $n \times m$ matrix $A^T = [a_{ij}^T]$ where $a_{ij}^T = a_{ji}$ ($1 \leq i \leq m, 1 \leq j \leq n$) is called the Transpose of A .

$$\text{Ex// } A = \begin{bmatrix} 4 & -2 & 3 \\ 0 & 5 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 6 & 2 & 4 \\ 3 & -1 & 2 \\ 0 & 4 & 3 \end{bmatrix} \quad C = [3 \ -5 \ 1]$$

Then

$$A^T = \begin{bmatrix} 4 & 0 \\ -2 & 5 \\ 3 & -2 \end{bmatrix} \quad B^T = \begin{bmatrix} 6 & 3 & 0 \\ 2 & -1 & 4 \\ 4 & 2 & 3 \end{bmatrix} \quad C^T = \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$$

$$1- (A)^T = A$$

$$2- (A+B)^T = A^T + B^T$$

$$3- (nA)^T = n.A^T$$

$$4- (A \cdot B)^T = A^T \cdot B^T$$

$$5- (ABC)^T = C^T \cdot B^T \cdot A^T$$