Composition of relations

Let A, B, C be sets and let :

$R : A \to B$ $\quad (R \subset A \times B)$

$S: B \to C$ $\quad (S \subset B \times C)$

There is a relation from A to C denoted by

$R \circ S$ (composition of R and S) : $A \to C$

$R \circ S = \{(a,c) : \exists b \in B$ for which $(a,b) \in R$ and $(b,c) \in S\}$

Example : let $A = \{1,2,3,4\}$

$B = \{a, b, c, d\}$

$C = \{x, y, z\}$

$R = \{(1,a),(2,d),(3,a),(3,d),(3,b)\}$

$S = \{(b,x),(b,z),(c,y),(d,z)\}$

Find $R \circ S$?

Solution :

1) The first way by arrow form