5. Tautology AND Contradictions

Some propositions P(p, q, . . .) contain only T in the last column of their truth tables or, in other words, they are true for any truth values of their variables. Such propositions are called **Tautologies**. Analogously, a proposition P(p, q, . . .) is called a **contradiction** if it contains only F in the last column of its truth table or, in other words, if it is false for any truth values of its variables. For example, the proposition "p or not p," that is, $p \vee \neg p$, is a tautology, and the proposition "p and not p," that is, $p \wedge \neg p$, is a contradiction.

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

(a) $p \vee \neg p$

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

(b) $p \wedge \neg p$

Note that the negation of a tautology is a contradiction since it is always false, and the negation of a contradiction is a tautology since it is always true.

Example:

Show that **$(p \wedge q) \vee \neg(p \wedge q)$** is a Tautology.

Solution:

The truth table for **$(p \wedge q) \vee \neg(p \wedge q)$** is given below

P	q	p∧q	¬(p∧q)	(p∧q)v ¬(p∧q)
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

The last column of the truth table contains only the truth value T and hence we can deduce that **$(p \wedge q) \vee \neg(p \wedge q)$** is a tautology.