vector space

Definition:- A vector space or linear space over the field F is an order trip ((V,+),(F,+,.),o) such that:

1)(V,+) is commutative group.

2)(F,+,.) is afield.

3) An operation o is called scalar multiplication satisfy the following:

a- if $c\in F$, $X\in V$ then $c \text{ o } v \in V$.

b- $(c_1+c_2)ox=c_1ox+c_2ox$. $\forall c_1,c_2\in F ,x\in V$

c- $(c_1.c_2)ox=c_1o(c_2ox)$.

d- $co(x+y)=cox+coy$. $\forall c\in F, x ,y\in V$

e- $1ox=x$ Where 1is identity of field.

Example:- Let $V=IR^n$ Defined the vector addition as $X+Y=(x_1+y_1),(x_2+y_2),\ldots,(x_n+y_n)$ defined Scalar multiplications

$\lambda oX=\lambda o(x_1,x_2,\ldots,x_n)=\lambda x_1,\lambda x_2,\ldots,\lambda x_n$

then $((IR^n,+),(R,+,.),o)$ is vector space $X\in IR^n\rightarrow X=(x_1,x_2,\ldots,x_n)$

1)$(IR^n,+)$ is comm-group.

2)(R,+,.) is field.

3)if $c\in F,x\in V$ then $cox\in V$

Let $\lambda\in F \rightarrow\lambda\in IR ,x\in IR^n$

$\therefore\ \lambda ox=\lambda o(x_1,x_2,\ldots,x_n)=\lambda x_1,\lambda x_2,\ldots,\lambda x_n \in IR^n$.

4)$(\lambda_1+\lambda_2)ox \quad \lambda_1,\lambda_2 \in \mathbb{R}$.

$=(\lambda_1+\lambda_2)o(x_1,x_2,\ldots,x_n)=\lambda_1 ox_1+\lambda_2 ox_2+\ldots\ldots+\lambda_1 ox_n+\lambda_2 ox_n$

$=(\lambda_1 x_1,\lambda_1 x_2,\ldots..,\lambda_1 x_n)+(\lambda_2 x_1,\ldots..,\lambda_2 x_n)=\lambda_1 ox+\lambda_2 ox$

5) $(\lambda_1.\lambda_2)ox=(\lambda_1.\lambda_2)o(x_1,x_2,\ldots\ldots,x_n)=((\lambda_1.\lambda_2).x_1,\ldots\ldots,(\lambda_1.\lambda_2).x_n)$

$=(\lambda_1.(\lambda_2.x_1),\ldots\ldots..,\lambda_1.(\lambda_2.x_n)=\lambda_1 o(\lambda_2 ox)$.

6)$\lambda o(x+y)=\lambda o(x_1+y_1,x_2+y_2,\ldots..,x_n+y_n)=(\lambda.(x_1+y_1),\lambda.(x_2+y_2),\ldots..,\lambda.(x_n+y_n))$

$=(\lambda.x_1+\lambda y_1,\ldots.,\lambda x_n+\lambda y_n)=(\lambda x_1,\lambda x_2,\ldots.,\lambda x_n)+(\lambda y_1,\lambda y_2,\ldots..,\lambda y_n)=\lambda ox+\lambda oy$.

7)$1ox=1o(x_1,x_2,\ldots.,x_n)=(1.x_1,1.x_2,\ldots.,1.x_n)=(x_1,x_2,\ldots.,x_n)=x$.

Remark:- A vector space V over the field F is called complex vector space if F is field of complex number and is called vector space if F is a field of Real number.

Examples:- 1) let x be set of all real function defined on straight line with addition

$(f+g)(x)=f(x)+g(x)$.

Scalar multiplication$(\alpha f)(x)=\alpha.f(x)$

Then x is vector space.

2)Let V the set of all m×n matrices with real entire then V is vector space where the operation is addition and multiplication.