Lecture 4

Balanced Motion Part 2

4.1 The Gradient Wind

The gradient wind is defined as the wind existing if the trajectory of a particle (or air parcel) is circular and we have a balance among the pressure gradient force, the Coriolis force and the centrifugal force.

A. Cyclonic flow (low pressure)

In this case, a Coriolis force and the centrifugal force act in the same direction. In order to have a balance, the pressure gradient force must act in the opposite direction and we have a low pressure in the center (see case a in Figure 4.1). If we take the effect of curvature into account, we have to expand the horizontal momentum equation to include the centrifugal term:

$$PGF = CF + CeF \qquad (4.1)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial n} = \mathrm{f\,V_G} + \frac{\mathrm{V_G^2}}{\mathrm{R}} \qquad (4.2)$$

and by using geostrophic balance $f\,V_g = -\frac{1}{\rho}\frac{\partial p}{\partial n}$, we substitute the left side in (4.2) by $\mathrm{f\,V_g}$:

$$\mathrm{f\,V_g} = \mathrm{f\,V_G} + \frac{\mathrm{V_G^2}}{\mathrm{R}} \qquad (4.3)$$

Here V_g is the geostrophic wind, V_G is the gradient wind, and R is the radius of curvature.

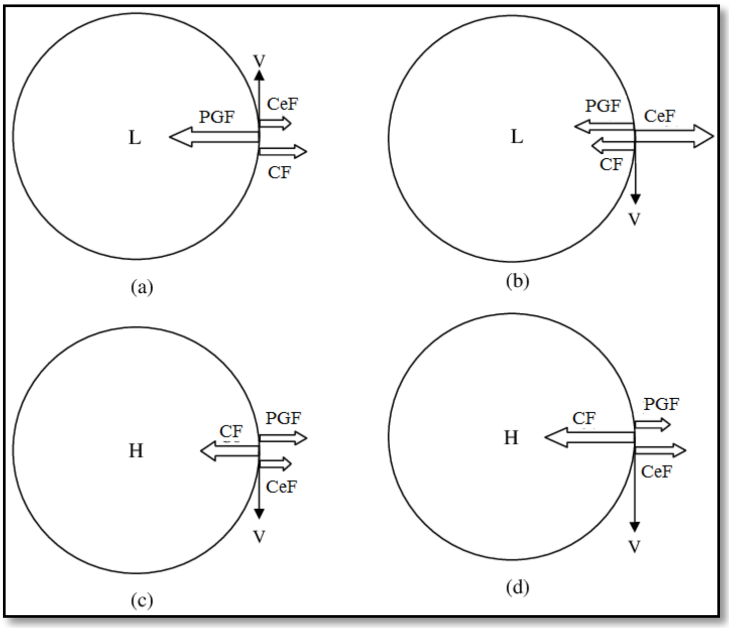


Fig. 4.1 Four balances for the four types of gradient flow.

The gradient wind speed is obtained by solving equation (4.3) for V_G to yield:

$$f\,V_g = f\,V_G + \frac{V_G^2}{R}$$

Dividing by V_G^2,

$$f\frac{V_g}{V_G^2} = \frac{f}{V_G} + \frac{1}{R}$$

$$fV_g\left(\frac{1}{V_G}\right)^2 - f\left(\frac{1}{V_G}\right) - \frac{1}{R} = 0$$

By using quadratic formula to solve,

$$x = \frac{-b \mp \sqrt{b^2 - 4\,a\,c}}{2\,a}$$

we get,

$$a = f\,V_g \qquad b = -f \qquad c = -\frac{1}{R} \qquad x = \frac{1}{V_G}$$

$$\frac{1}{V_G} = \frac{f \mp \sqrt{f^2 + 4\,\frac{f\,V_g}{R}}}{2\,f\,V_g}$$

Dividing the numerator and the denominator of the right side on $(2\,f)$ we get,

$$\frac{1}{V_G} = \frac{\frac{1}{2} \mp \sqrt{\frac{1}{4} + \frac{V_g}{R\,f}}}{V_g}$$

$$\therefore \qquad V_G = \frac{V_g}{\frac{1}{2} \mp \sqrt{\frac{1}{4} + \frac{V_g}{R\,f}}}$$

This equation tells us that $V_G < V_g$ in all cases because the denominator is larger than one. The difference between V_G & V_g becomes larger at smaller R, and at smaller latitude angle. To illustrate this difference we consider:

At $V_g = 10\frac{m}{s}$ $\quad and\ \ latitude = 45^o$

$if \quad R = 1000\,km$, we find $V_G = 9.18\,m/s$ and the difference between V_G & V_g is small.

When R becomes much smaller the difference between V_G & V_g will be large (for example at $R = 10\ km,\ V_G = 2.73\,m/s$) .

If we assume that $latitude = 45^o\ and\ \ V_g = 10\frac{m}{s}$ we may calculate the value of R necessary to make $V_G = \frac{1}{2}V_g$, we find from the equation that the radius of $R = 50\ km.$

See Table (4.1) for more details.

Table (4.1) The gradient wind speed at latitude 45° and $V_g = 10\,m/s$ at different R values for law pressure

No.	R (m)	denominator only at (+) case	square root only	V_G at (+) case	V_G at (-) case	V_G at (+) case at latitude 30°
1	1000	10.36003100	9.860031	0.965	-106.84	0.818250086
2	10000	3.653889842	3.153890	2.737	-3768.05	2.360273054
3	50000	1.979663552	1.479664	5.051	-51037.93	4.484402714
4	100000	1.604401247	1.104401	6.233	-165453.00	5.639112600
5	500000	1.166288543	0.666289	8.574	-3006821.70	8.169486737
6	**1000000**	**1.089041774**	**0.589042**	**9.182**	**-11230683.72**	**8.911043629**
7	2000000	1.046337904	0.546338	9.557	-43161209.60	9.394800613
8	4000000	1.023681729	0.523682	9.769	-168906589.24	9.678827156

B. Anticyclonic flow (high pressure)

In this case, a pressure gradient force and the centrifugal force are in the same direction. In order to have a balance the Coriolis force must act in the opposite direction, we have a high pressure in the center (case c and d in Fig. 4.1).

$$PGF + Ce\ F - CF = 0$$

$$f\ V_g + \frac{V_G^2}{R} - f\ V_G = 0$$

In the same previous manner,

$$\therefore \quad V_G = \frac{V_g}{\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{V_g}{R\,f}}}$$

We see that $V_G > V_g$ in all cases.

In the special case where $\frac{V_g}{R\,f} = \frac{1}{4}$, $V_G = 2V_g$, the maximum wind in the anticyclonic case is therefore twice the geostrophic wind and hence if we assume that, $f = 10^{-4}\ s^{-1}$ and $V_g = 10\,m/s$, the radius of curvature is equal to about 400 km, which is quite small.

Table (4.2) The gradient wind speed at latitude 45° and $V_g = 10\ m/s$ at different R values for high pressure

No.	R (m)	V_G
1	1000	-
2	10000	-
3	50000	-
4	100000	-
5	387881	19.98738189
6	500000	13.57277449
7	1000000	11.22094898
8	2000000	10.53847271
9	4000000	10.25494409

4.2 The Cyclostrophic Flow

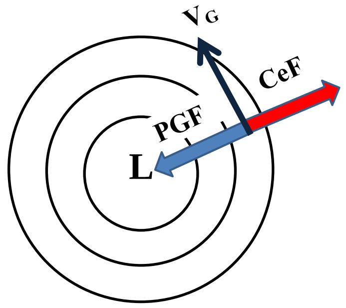
Cyclostrophic balance occurs when the pressure gradient force and centrifugal force are equal and in opposite direction. This is the situation near the equator

$$GF = Ce\ F$$

$$f\ V_g = \frac{V_G^2}{R}$$

$$V_G^2 = f\ V_g R$$

$$\therefore \quad V_G = \sqrt{f\ V_g R}$$



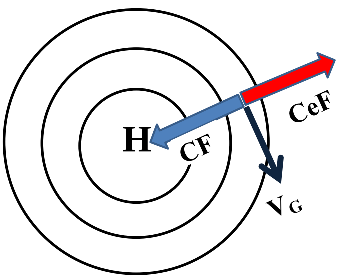
4.3 The Inertial Flow

In inertial flow, there is no pressure gradient force, there are two forces only, Coriolis and centrifugal that may balance each other.

$$CF = Ce\ F$$

$$f\ V_G = \frac{V_G^2}{R}$$

$$V_G = Rf$$



Gradient Wind and Geostrophic Wind

Lecture Notes – Atmospheric Dynamics II

1. Introduction

The gradient wind describes atmospheric motion along curved isobars. It represents a balance between three forces acting on an air parcel: the pressure gradient force, the Coriolis force, and the centrifugal force. This concept extends the geostrophic balance by including curvature effects.

2. Gradient Wind Equation

$f V_g + (V_g^2 / R) = (1/\rho) (\partial p/\partial n)$

Where:

V_g : Gradient wind speed

f : Coriolis parameter

R : Radius of curvature of the flow

ρ : Air density

$\partial p/\partial n$: Pressure gradient perpendicular to the flow

3. Forces Acting on the Air Parcel

3.1 Pressure Gradient Force (PGF)

The pressure gradient force drives air from regions of high pressure toward regions of low pressure and is the primary force initiating atmospheric motion.

3.2 Coriolis Force

The Coriolis force results from the rotation of the Earth. It deflects moving air to the right in the Northern Hemisphere and to the left in the Southern Hemisphere.

3.3 Centrifugal Force

The centrifugal force appears when air follows a curved trajectory. It depends on both the wind speed and the radius of curvature of the flow.

4. Cyclonic Flow (Low Pressure System)

In the Northern Hemisphere, air circulates counterclockwise around a low-pressure system.

Force directions:

Pressure Gradient Force → toward the center

Coriolis Force → outward

Centrifugal Force → outward

Force balance:

PGF = Coriolis + Centrifugal

Result:

The gradient wind speed is smaller than the geostrophic wind speed (Vg < VG).

Conceptual Diagram (Cyclone)

```
       Coriolis
          ↑
          |
PGF  -----> L <-----  PGF
          |
          ↓
     Centrifugal
```

5. Anticyclonic Flow (High Pressure System)

In the Northern Hemisphere, air circulates clockwise around a high-pressure system.

Force directions:

Pressure Gradient Force → outward

Coriolis Force → inward

Centrifugal Force → outward

Force balance:

PGF + Centrifugal = Coriolis

Result:

The gradient wind speed is greater than the geostrophic wind speed (Vg > VG).

Conceptual Diagram (Anticyclone)

```
    Centrifugal
        ↑
        |
PGF -----> H -----> PGF
        |
        ↓
     Coriolis
```

6. Geostrophic Wind

$$VG = (1 / \rho f) (\partial p/\partial n)$$

The geostrophic wind occurs when the isobars are straight ($R \rightarrow \infty$). In this case the centrifugal force disappears and the balance is only between the pressure gradient force and the Coriolis force.

7. Key Comparison

Feature	Geostrophic Wind	Gradient Wind
Isobars	Straight	Curved
Forces	PGF + Coriolis	PGF + Coriolis + Centrifugal
Cyclone relation	VG > Vg	Vg < VG
Anticyclone relation	VG < Vg	Vg > VG

8. Important Notes for Students

1. The gradient wind is more realistic in atmospheric flows where curvature exists.
2. The difference between gradient wind and geostrophic wind increases when the radius of curvature decreases.
3. The difference also increases at lower latitudes where the Coriolis parameter becomes smaller.
4. The concept is widely used in synoptic meteorology to understand the circulation around cyclones and anticyclones.