2.5.2 Minimizing Distance Method (MDM)

Example (2.2): (www.neuroptics.com)

Lets have the following TSP

	A	B	C	D	E
A	--	7	6	8	4
B	7	--	8	5	6
C	6	8	--	9	7
D	8	5	9	--	8
E	4	6	7	8	--

1st step: we have to minimize row and column.

	A	B	C	D	E
A	--	3	2	4	0
B	2	--	3	0	1
C	0	2	--	3	1
D	3	0	4	--	3
E	0	2	3	4	--

$\Rightarrow$

	A	B	C	D	E
A	--	3	0	4	0
B	2	--	1	0	1
C	0	2	--	3	1
D	3	0	2	--	3
E	0	2	1	4	--

2nd step: calculate penalties of all 0's (least no. in row + least no. in column). All rows and columns have at least one zero.

	A	B	C	D	E
A	--	3	0^{0+1}	4	0^{0+1}
B	2	--	1	0^{3+1}	1
C	0^{1+0}	2	--	3	1
D	3	0^{2+2}	2	--	3
E	0^{1+0}	2	1	4	--

Choose max Penalty if Equal choose arbitrarily

$\Rightarrow$

	A	B	C	D	E
A	--	3	0^{1}	4	0^{1}
B	2	--	2	0^{4}	1
C	0^{1}	2	--	3	1
D	3	0^{4}	2	--	3
E	0^{1}	2	1	4	--

Max penalty is 4.

Cross row of B and column of D. we have **B→D** route. So we have the following reduced matrix.

	A	B	C	E
A	--	3	0	0
C	0	2	--	1
D	3	0	2	3
E	0	2	1	--

⇒

	A	B	C	E
A	--	3	0	0
C	0	2	--	1
D	3	--	2	3
E	0	2	1	--

Since we obtained the route **B→D** then we change the cell D-B to (--).

Now we will reduced the matrix by taking row and column min.

	A	B	C	E
A	--	3	0	0
C	0	2	--	1
D	1	--	0	1
E	0	2	1	--

⇒

	A	B	C	E
A	--	1	0^0	0^1
C	0^0	0^0	--	1
D	1	--	0^1	1
E	0^0	0^0	1	--

Max. penalty is 1. Choose **A→E** arbitrarily.

Now reduce matrix again:

	A	B	C
C	0	0	--
D	1	--	0
E	0	0	1

⇒

	A	B	C
C	0^1	0^0	--
D	1	--	0^2
E	--	0^1	1

We have at least one zero in each row and column. Max. penalty is 2. We have **D→C**, reduced matrix.

	A	B
C	0^0	0^0
E	--	0^0

Here we can't choose **E→A** (then can't choose **C→B**), so we have to choose **E→B**.

Lastly we have **C→A**.

So we have the following routs:

B→D, A→E, D→C, E→B and **C→A**.

Then we obtain the following complete route:

A→E→B→D→C→A, with total minimum distance:

Z=4+6+5+9+6 = 30.

Exercises (2.1):

1.

	A	B	C	D	E
A	--	3	6	2	3
B	3	--	5	2	3
C	6	5	--	6	4
D	2	2	6	--	6
E	3	3	4	6	--

2.

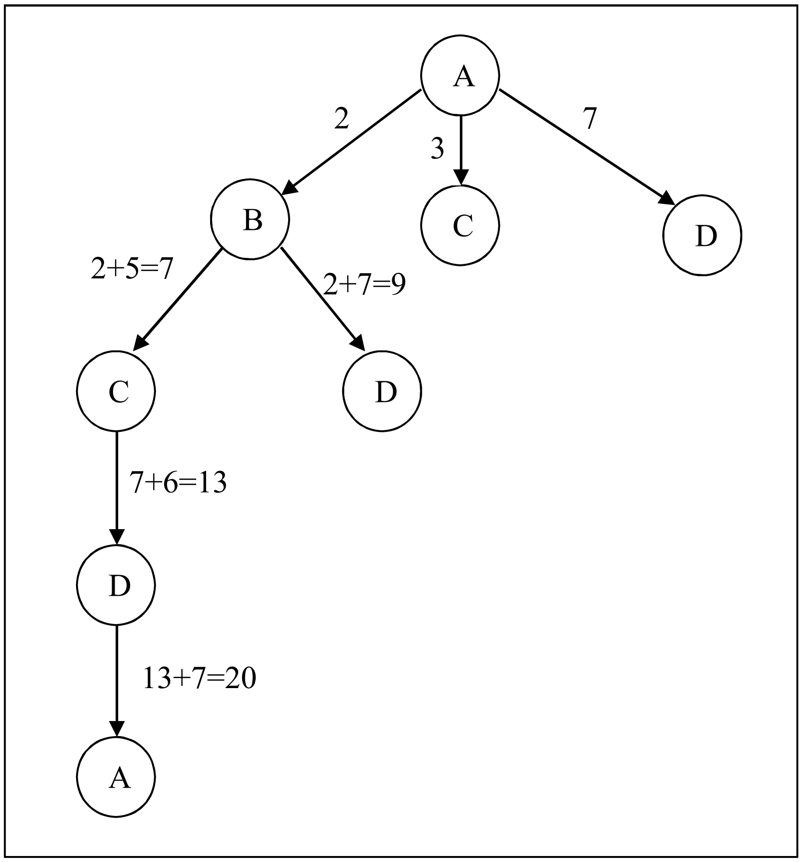
	A	B	C	D	E	F
A	--	20	23	27	29	34
B	21	--	19	26	31	24
C	26	28	--	15	36	26
D	25	16	25	--	23	18
E	23	40	13	31	--	10
F	27	18	12	35	16	--

2.5.3 Tree Type Heuristic Method (TTHM)

The main step in this method, the objective function is evaluated at all nodes in each level of the search tree, then some of the nodes within each level (with minimum or maximum value) of the search tree are chosen from which to branch.

Usually, one node is chosen with each level and stop at the first complete sequence of the nodes to be the solution.

Example (2.3): Call example (2.1)



The route is : A-B-C-D-A with Z=20.

Exercises (2.2):

1. A-C-D-B-A, Z=15.

	A	B	C	D
A	--	3	3	7
B	3	--	5	5
C	3	5	--	4
D	7	5	4	--

2. A-B-D-C-E-A, Z=90.

	A	B	C	D	E
A	--	15	18	15	21
B	19	--	18	17	20
C	14	20	--	19	20
D	24	23	20	--	22
E	18	21	23	25	--

2.5.4 Dynamic Programming / Held Karp Algorithm

http://www.facebook.com/tusharroy25

Dynamic programming (DP) is a mathematical technique well suited for the optimization of multistage decision problems. The DP for n-variables problem is represented as a sequence of n single variable problems which are solved successively. The main objective of DP is finding the shortest path in a state space graph G in which vertices corresponding to subset S and in which arcs corresponding to a decision where by the transition to a new state from a previous state is archived by sequencing the nodes. Clearly, there are 2^n vertices in the graph.

Example (2.4): Solve the following TSP using DP.

	A	B	C	D
A	--	1	15	6
B	2	--	7	3
C	9	6	--	12
D	10	4	8	--

Solution:

Start at vertex=A

$2^n=2^3$ status:

1. {A}.
2. {B},{C},{D}.

3. {B,C},{B,D},{C,D}.
4. {B,C,D}.
1. {A}

		Cost	Parent
[A,B]	=	1	A
[A,C]	=	15	A
[A,D]	=	6	A

2. {B},{C},{D}

						Cost	Parent
[{B},D]	=	[A,B]+B→C					
	=	B→C+[A,B]	=	7+1	=	8	B
[{B},D]	=	B→D+[A,B]	=	3+1	=	4	B
[{C},B]	=	C→B+[A,C]	=	6+15	=	21	C
[{C},D]	=	C→D+[A,C]	=	12+15	=	27	C
[{D},B]	=	D→B+[A,D]	=	4+6	=	10	D
[{D},C]	=	D→C+[A,D]	=	8+6	=	14	D

3. {B,C},{B,D},{C,D}

				Cost	Parent
[{B,C},D]	=	min{B→D+[{C},B], C→D+[{B},C]}			
	=	min {3+21,12+8}	=	20	C
[{B,D},C]	=	min{B→C+[{D},B], D→C+[{B},D]}			
	=	min {7+10,8+4}	=	12	D
[{C,D},B]	=	min{C→B+[{D},C], D→B+[{C},D]}			
	=	min {6+14,4+27}	=	20	C

4. {B,C,D}

$[\{B,C,D\},A] = \min\{B \to A+[\{C,D\},B],\ C \to A+[\{B,D\},C],\ D \to A+[\{B,C\},D]\}$

$= \min\{2+20, 9+12, 10+20\} \qquad = 21 \qquad C$

The route is: **A→B→D→C→A**, with Z=1+3+8+9=21.

Time complexity: since we have 2^n subsets and matrix $n \times n = n^2$, then the complexity is $O(2^n \times n^2)$.

Exercises (2.3):

1. A-D-B-C-E-A, Z=16.

	A	B	C	D	E
A	--	3	6	2	3
B	3	--	5	2	3
C	6	5	--	6	4
D	2	2	6	--	6
E	3	3	4	6	--

2. A-E-F-C-D-B-A, Z=103.

	A	B	C	D	E	F
A	--	20	23	27	29	34
B	21	--	19	26	31	24
C	26	28	--	15	36	26
D	25	16	25	--	23	18
E	23	40	13	31	--	10
F	27	18	12	35	16	--