3. prove $P = \rho R_d T_v$

$$\rho = \rho_d' + \rho_v' \qquad (1)$$

we have $e = \rho_v' R_v T$ so $\Longrightarrow$ $\rho_v' = \frac{e}{R_v T}$ ....(2)

and we have $P_d' = \rho_d' R_d T$ so $\Longrightarrow$ $\rho_d' = \frac{P_d'}{R_d T}$ ....(3)

Substituting equation (2 and 3) into (1) we obtain

$$\rho = \frac{P_d'}{R_d T} + \frac{e}{R_v T} \quad \ldots\ldots\ldots..(4)$$

$$\because \; P = P_d' + e \quad so \;\Longrightarrow\; P_d' = P - e \;\ldots\ldots\ldots(5)$$

Substituting equation (5) into (4) we obtain

$$\rho = \frac{P - e}{R_d T} + \frac{e}{R_v T} \qquad \ldots\ldots\ldots..(6)$$

we have $\frac{R_d}{R_v} = \varepsilon$ so $R_v = \frac{R_d}{\varepsilon}$

$$\rho = \frac{P - e}{R_d T} + \frac{e}{\frac{R_d}{\varepsilon}\; T} \qquad \Longrightarrow \rho = \frac{P - e}{R_d T} + \frac{\varepsilon\, e}{R_d\; T}$$

$$\Longrightarrow \rho = \frac{P}{R_d T} - \frac{e}{R_d T} + \frac{\varepsilon\, e}{R_d\; T}$$

$$\rho = \frac{P}{R_d T}\left[1 - (\frac{e}{P} + \frac{\varepsilon e}{P})\right] \quad \Longrightarrow \rho = \frac{P}{R_d T}\left[1 - \frac{e}{P}(1 - \varepsilon)\right]$$

$$P\left[1 - \frac{e}{P}(1 - \varepsilon)\right] = \rho R_d T$$

$$P = \rho R_d \frac{T}{\left[1 - \frac{e}{P}(1 - \varepsilon)\right]}$$

$$T_v = \frac{T}{1 - \frac{e}{P}(1 - \varepsilon)}$$

$$P = \rho R_d T_v$$