# Foundation of Mathematics 1

# CHAPTER 2 SETS

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# Chapter Two
# Sets

## 2.1. Definitions

**Definition 2.1.1.** A **set** is a collection of (objects) things. The things in the collection are called **elements (member)** of the set.

A set with no elements is called **empty set** and denoted by $\emptyset$; that is, $\emptyset = \{\ \}$.
A set that has only one element, such as $\{x\}$, is sometimes called a **singleton set**.

List of the symbols we will be using to define other terminologies:
$|$ **or :** : such that
$\in$ : an element of (belong to)
$\notin$ : not an element of (not belong to)
$\subset$ **or** $\subsetneq$: a proper subset of
$\subseteq$ : a subset of
$\nsubseteq$ : not a subset of
$\mathbb{N}$ : Set of all natural numbers
$\mathbb{Z}$ : Set of all integer numbers
$\mathbb{Z}^+$ : Set of all positive integer numbers
$\mathbb{Z}^-$ : Set of all negative integer numbers
$\mathbb{Z}_o$ : Set of all odd numbers
$\mathbb{Z}_e$ : Set of all even numbers
$\mathbb{Q}$ : Set of all rational numbers
$\mathbb{R}$ : Set of all real numbers

**Set Descriptions 2.1.2.**

**(i) Tabulation Method**
The elements of the set listed between commas, enclosed by braces.
(1) $\{1,2,37,88,0\}$
(2) $\{a, e, i, o, u\}$ Consists of the lowercase vowels in the English alphabet.
(3) $\{\dots, -4, -2, 0, 2, 4, 6\}$ Continue from left side
$\{-4, -2, 0, 2, 4, 6, \dots\}$ Continue from right side
$\{\dots, -4, -2, 0, 2, 4, 6, \dots\}$ Continue from left and right sides.
(4) $B = \{\{2,4,6\}, \{1,3,7\}\}$.

**(ii) Rule Method**

Describe the elements of the set by listing their properties writing as

$$S = \{x|\ A(x)\},$$

where $A(x)$ is a statement related to the elements $x$. Therefore,

$$\text{x} \in S \Leftrightarrow A(x) \text{ is hold}$$

(1) $A = \{x|x \text{ is a positive integers and } x > 10\}$
$A = \{x|x \ \in \mathbb{Z}^{+} \text{and } x > 10\}.$
(2) $\mathbb{Z}_o = \{x|x = 2n - 1 \text{ and } n \ \in \mathbb{Z}\}$
$= \{2n - 1|\ n \ \in \mathbb{Z}\}.$
(3) $\{x \in \mathbb{Z}|\ |x| < 4\} = \{-3, -2, -1,0,1,2,3\}.$
(4) $\{x \in \mathbb{Z}|\ x^2 - 2 = 0\} = \emptyset.$

**Examples 2.1.3.**

**(i)** $\mathbb{Z} = \{\dots, -3, -2, -1,0,1,2,3, \dots\}$ Integer numbers.
**(ii)** $\mathbb{Z}_e = \{x|x = 2n \text{ and } n \ \in \mathbb{Z}\}$
$= \{2n|\ n \ \in \mathbb{Z}\}.$ Even numbers
Note that 2 is an element of $\mathbb{Z}_e$ so, we write $2 \ \in \ \mathbb{Z}$. But, $5 \notin \ \mathbb{Z}_e$.

**(iii)** Let $C$ be the set of all natural numbers which are less than 0.
In this set, we observe that there are no elements. Hence, $C$ is an empty set; that is,

$$C = \emptyset.$$

**Definition 2.1.4.**

**(i)** A set $A$ is said to be a **subset** of a set $B$ if every element of $A$ is an element of $B$ and denote that by $A \subseteq B$. Therefore,

$$A \subseteq B \Leftrightarrow \forall x(x \in A \Longrightarrow x \in B).$$

**(ii)** If $A$ is a nonempty subset of set $B$ and $B$ contains an element which is not a member of $A$, then $A$ is said to be **proper subset** of $B$ and denoted this by $A \ \subset \ B$ or $A \ \subsetneq \ B$; that is, $A$ is said to be a **proper subset** of $B$ if and only if
(1) $A \ \neq \ \emptyset$, (2) $A \ \subset \ B$ and (2) $A \ \neq \ B$.

We use the expression $A \ \nsubseteq \ B$ means that $A$ is **not** a subset of $B$.

**Examples 2.1.5.**

**(i)** An empty set $\emptyset$ is a subset of any set $B$; that is, for every set $B$, $\phi \subseteq B$.

If this were not so, there would be some element $x \ \in \ \emptyset$ such that $x \ \notin \ B$. However, this would contradict with the definition of an empty set as a set with no elements.

**(ii)** Let $B$ be the set of natural numbers. Let $A$ be the set of even natural numbers. Clearly, $A$ is a subset of $B$. However, $B$ is not a subset of $A$, for $3 \in B$, but $3 \notin A$.

**Theorem 2.1.6. (Properties of Sets)**
Let $A, B$, and $C$ be sets.
**(i)** For any set $A, A \subseteq A$. (Reflexive Property)
**(ii)** If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$. (Transitive Property)

**Proof.**
**(ii)**

| | | |
|---|---|---|
| 1 | $(A \subseteq B) \Leftrightarrow \forall x(x \in A \Rightarrow x \in B)$ | Hypothesis and Def. $\subseteq$ |
| 2 | $(B \subseteq C) \Leftrightarrow \forall x(x \in B \Rightarrow x \in C)$ | Hypothesis and Def. $\subseteq$ |
| | $\Rightarrow \forall x(x \in A \Rightarrow x \in C)$ | Inf. (1),(2) Syllogism Law |
| | $\Leftrightarrow A \subseteq C$ | Def. of $\subseteq$ |

**Definition 2.1.7** If $X$ is a set, the **power set** of $X$ is another set, denoted as $P(X)$ and defined to be the set of all subsets of $X$. In symbols,

$$P(X) = \{A | A \subseteq X\}.$$

That is, $A \subseteq X$ if and only if $A \in P(X)$.

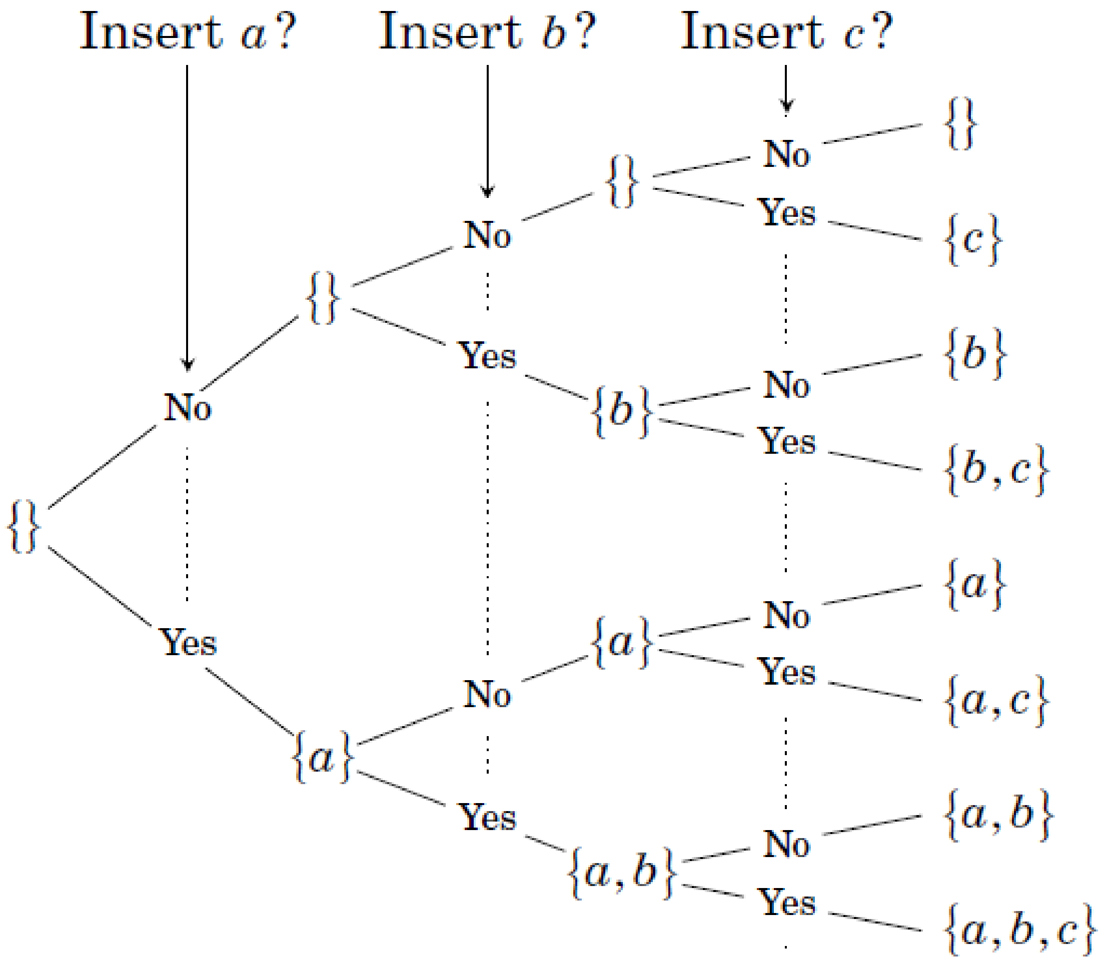
**Example 2.1.8.**

**(i)** $\emptyset$ and a set $X$ are always members of $P(X)$.

**(ii)** suppose $X = \{a, b, c\}$. Then

$$P(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}.$$

The way to finding all subsets of $X$ is illustrated in the following figure.



From the above example, if a finite set $X$ has $n$ elements, then it has $\mathbf{2^n}$ subsets, and thus its power set has $\mathbf{2^n}$ elements.

**(iii)** $P(\{1,2,4\}) = \{\emptyset, \{0\}, \{1\}, \{4\}, \{0,1\}, \{0,4\}, \{1,4\}, \{1,2,4\}\}$.
**(iv)** $P(\emptyset) = \{\emptyset\}$.
**(v)** $P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$.
**(vi)** $P(\{\mathbb{Z}, \mathbb{R}\}) = \{\emptyset, \{\mathbb{Z}\}, \{\mathbb{R}\}, \{\mathbb{Z}, \mathbb{R}\}\}$.

The following are wrong statements.

**(vii)** $P(1) = \{\emptyset, \{1\}\}$.
**(viii)** $P(\{1, \{1,2\}\}) = \{\ \emptyset, \{1\}, \{1,2\}, \{1, \{1,2\}\}\ \}$.
**(ix)** $P(\{1, \{1,2\}\}) = \{\ \emptyset, \{\{1\}\}, \{\{1,2\}\}, \{1, \{1,2\}\}\ \}$.

## 2.2. Equality of Sets

**Definition 2.2.1.** Two sets, $A$ and $B$, are said to be **equal** if and only if $A$ and $B$ contain exactly the same elements and denote that by $A = B$. That is, $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
The description $A \neq B$ means that $A$ and $B$ are not equal sets.

**Example 2.2.2.**
Let $\mathbb{Z}_e$ be the set of even integer numbers and $B = \{x | x \in \mathbb{Z}$ and divisible by $2\}$.
Then $\mathbb{Z}_e = B$.
Proof.
To prove $\mathbb{Z}_e \subseteq B$.
$\mathbb{Z}_e = \{2n | n \in \mathbb{Z}\}$.
$x \in \mathbb{Z}_e \Leftrightarrow \exists\, n \in \mathbb{Z} : x = 2n$ Def. of $\mathbb{Z}_e$.
$\Longrightarrow \frac{x}{2} = n$ Divide both side of $x = 2n$ by 2.
$\Longrightarrow x \in B$ Def. of $B$.
(1) $\Longrightarrow \mathbb{Z}_e \subseteq B$ Def. of subset.
To prove $B \subseteq \mathbb{Z}_e$.
$x \in B \Leftrightarrow \exists\, n \in \mathbb{Z} : \frac{x}{2} = n$ Def. of $\mathbb{Z}_e$.
$\Longrightarrow x = 2n$ Multiply $\frac{x}{2} = n$ by 2.
$\Longrightarrow x \in \mathbb{Z}_e$ Def. of $\mathbb{Z}_e$.
(2) $\Longrightarrow B \subseteq \mathbb{Z}_e$ Def. of subset.

$\mathbb{Z}_e = B$ inf (1),(2) and def. of equality.

**Remark 2.2.3.**
**(i)** Two equal sets always contain the same elements. However, the rules for the sets may be written differently, as in **Example 2.2.2**.
**(ii)** Since any two empty sets are equal, therefore, there is a unique empty set.
**(iii)** the symbols $\subseteq, \subset, \subsetneq, \nsubseteq$ are used to show a relation between two sets and not between an element and a set. With one exception, if $x$ is a member of a set $A$, we may write $x \in A$ or $\{x\} \subseteq A$, but **not** $x \subseteq A$.
**(iv)** $\phi \neq \{\phi\}$.

**Theorem 2.2.4. (Properties of Set Equality)**
**(i)** For any set $A$, $A = A$. (Reflexive Property)
**(ii)** If $A = B$, then $B = A$. (Symmetric Property)
**(iii)** If $A = B$ and $B = C$, then $A = C$. (Transitive Property)

**Definition 2.2.5.** Let $A$ and $B$ be subsets of a set $X$. The **intersection** of $A$ and $B$ is the set

$$A \cap B = \{x \in X \mid x \in A \text{ and } x \in B\},$$

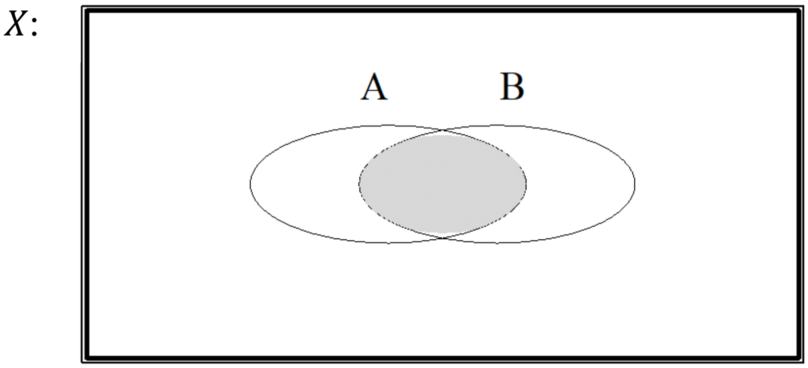
or

$$A \cap B = \{x \in X \mid x \in A \wedge x \in B\}.$$

Therefore, $A \cap B$ is the set of all elements in common to both $A$ and $B$.

**Example 2.2.6.**

**(i)** Given that the box below represents $X$, the shaded area represents $A \cap B$:



**(ii)** Let $A = \{2,4,5\}$ and $B = \{1,4,6,8\}$. Then, $A \cap B = \{4\}$.

**(iii)** Let $A = \{2,4,5\}$ and $B = \{1,3\}$. Then $A \cap B = \emptyset$.

**Definition 2.2.7.** If two sets, $A$ and $B$ are two sets such that $A \cap B = \emptyset$ we say that $A$ and $B$ are **disjoint.**

**Definition 2.2.8.** Let $A$ and $B$ be two subsets of a set $X$. The **union** of $A$ and $B$ is the set

$$A \cup B = \{x \in X \mid x \in A \text{ or } x \in B\},$$

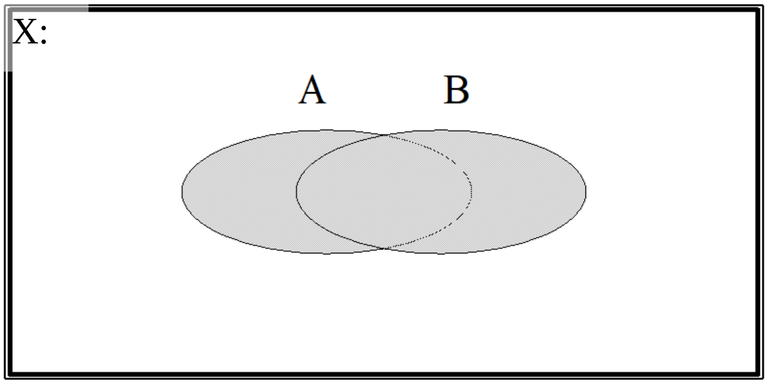
or

$$A \cup B = \{x \in X \mid x \in A \vee x \in B\}.$$

Therefore, $A \cup B$ = the set of all elements belonging to $A$ or $B$.

**Example 2.2.9.**

**(i)** Given that the box below represents $X$, the shaded area represents $A \cup B$:



**(ii)** Let $A = \{2,4,5\}$ and $B = \{1,4,6,8\}$. Then, $A \cup B = \{1,2,4,5,6,8\}$.
**(iii)** $\mathbb{Z}_e \cup \mathbb{Z}_o = \mathbb{Z}$.

**Remark 2.2.10.**
It is easy to extend the concepts of intersection and union of two sets to the intersection and union of a finite number of sets. For instance, if $X_1, X_2, \ldots, X_n$ are sets, then

$$X_1 \cap X_2 \cap \ldots \cap X_n = \{x | x \in X_i \text{ for all } i = 1, \ldots, n\}$$

and

$$X_1 \cup X_2 \cup \ldots \cup X_n = \{x \mid x \in X_i \text{ for some } i = 1,2, \ldots, n\}.$$

Similarly, if we have a collection of sets $\{X_i : i = 1,2, \ldots\}$ indexed by the set of positive integers, we can form their intersection and union. In this case, the intersection of the $X_i$ is

$$\bigcap_{i=1}^{\infty} X_i = \{x \in X_i \text{ for all } i = 1,2, \ldots\},$$

and the union of the $X_i$ is

$$\bigcup_{i=1}^{\infty} X_i = \{x \in X_i \text{ for some } i = 1,2, \ldots\}.$$

**Theorem 2.2.11.** Let $A, B$, and $C$ be arbitrary subsets of a set $X$. Then

**(i)** $A \cap B = B \cap A$ (Commutative Law for Intersection)

$A \cup B = B \cup A$ (Commutative Law for Union)

**(ii)** $A \cap (B \cap C) = (A \cap B) \cap C$ (Associative Law for Intersection)
$A \cup (B \cup C) = (A \cup B) \cup C$ (Associative Law for Union)
**(iii)** $A \cap B \subseteq A$
**(iv)** $A \cap X = A$; $A \cup \emptyset = A$
**(v)** $A \subseteq A \cup B$
**(vi)** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (Distributive Law of Union with respect to Intersection).
**(vii)** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive Law of Intersection with respect to Union),
**(viii)** $A \cup A = A, A \cap A = A$ (Idempotent Laws)
**(ix)** $A \cup \emptyset = A, A \cap X = A$ (Identity Laws)
**(x)** $A \cup X = X,\ A \cap \emptyset = \emptyset$ (Domination Laws)
**(xi)** $A \cup (A \cap B) = A$ (Absorption Laws)
$A \cap (A \cup B) = A$.

**Proof.**
**(i)** $A \cap B = B \cap A$. This proof can be done in two ways.

***The first proof***
Uses the fact that the two sets will be equal only if
$(A \cap B) \subseteq (B \cap A)$ and $(B \cap A) \subseteq (A \cap B)$.

(1) Let $x$ be an element of $A \cap B$

| | |
|---|---|
| Therefore, $x \in A \wedge x \in B$ | Def. of $\cap$ |
| Thus, $x \in B \wedge x \in A$ | Commutative Property of $\wedge$ |
| Hence, $x \in B \cap A$ | Def. of $B \cap A$ |
| Therefore, $A \cap B \subseteq B \cap A$ | Def. of $\subseteq$ |

(2) Let $x$ be an element of $B \cap A$

| | |
|---|---|
| Therefore, $x \in B \wedge x \in A$ | Def. of $\cap$ |
| Thus, $x \in A \wedge x \in B$ | Commutative property of $\wedge$ |
| Hence, $x \in A \cap B$ | Def. of $\cap$ |
| Thus, $B \cap A \subseteq A \cap B$ | Def. of $\subseteq$ |

Therefore, $A \cap B = B \cap A$ Inf. (1),(2)

***The second proof***

$A \cap B = \{x \mid x \in A \cap B\}$
$= \{x \mid x \in A \wedge x \in B\}$ Def. of $\cap$
$= \{x \mid x \in B \wedge x \in A\}$ Commutative property of $\wedge$
$= \{x \mid x \in B \cap A\}$ Def, of $\cap$
$= B \cap A$

**(iii)** $(A \cap B) \subseteq A$

It must be shown that each element of $A \cap B$ is an element of $A$.

Let $x \in A \cap B$
Thus, $x \in A \wedge x \in B$ Def. of $\cap$
Hence, $x \in A$

Therefore, $(A \cap B) \subseteq A$

**(iv)** $A \cap X = A$

(1) $A \cap X \subset A$ Inf. (iii) above

(2) Let $x \in A$
Thus, $x \in X$ $A \subseteq X$ is given
Hence, $x \in A \wedge x \in X$ Def. of $\wedge$
Therefore, $x \in A \cap X$ Def. of $\cap$
Thus, $A \subseteq A \cap X$ Def. of $\subseteq$

Thus, $A \cap X = A$ Inf. (1),(2)

**Definition 2.2.12.** Let $A$ and $B$ be subsets of a set $X$. The set $B - A$, called the **difference** of $B$ and $A$, is the set of all elements in $B$ which are not in $A$.
Thus,

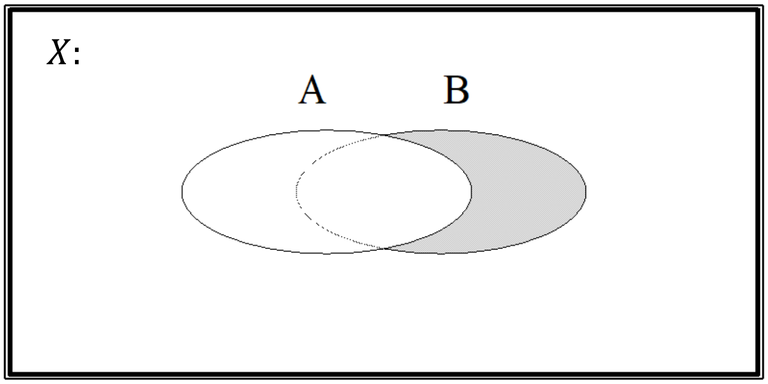
$$B - A = \{x \in X \mid x \in B \text{ and } x \notin A\}.$$

**Example 2.2.13.**

**(i)** Let $B = \{2,3,6,10,13,15\}$ and $A = \{2,10,15,21,22\}$.Then
$$B - A = \{3,6,13\}.$$

**(ii)** $\mathbb{Z} - \mathbb{Z}_o = \mathbb{Z}_e$.

**(iii)** Given that the box below represents $X$, the shaded area represents $B - A$.



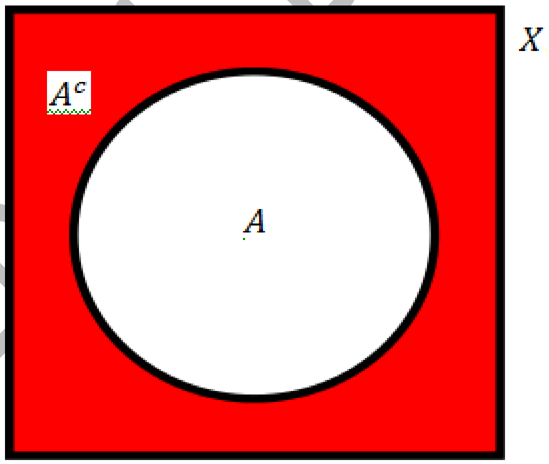
**Theorem 2.2.14.** Let $A$ and $B$ be subsets of a set $X$. Then

**(i)** $A - A = \emptyset,\ A - \emptyset = A$ and $\emptyset - A = \emptyset$

**Definition 2.2.15.** If $A \subseteq X$, then $X - A$ is called the **complement** of $A$ with respect to $X$ and denoted that by the symbol

$$X \backslash A \text{ or } A^c.$$

Thus, $A^c = \{x \in X \mid x \notin A\}$.



**Theorem 2.2.16.** Let $A$ and $B$ be subsets of a set $X$. Then

**(i)** $A^{c^c} = A$.

**(ii)** $X^c = \emptyset;\ \emptyset^c = X$.

**(iii)** $A \cup A^c = X,\ A \cap A^c = \emptyset$ (Inverse Laws)

**(iv)** If $A \subseteq B$, then $B^c \subseteq A^c$.

**(v)** $A \cap B = \emptyset \Leftrightarrow A \subseteq B^c$.

**Proof. Exercise.**

**Theorem 2.2.17.** Let $A$ and $B$ be subsets of a set $X$. Then

**(i)** $\left.\begin{aligned}(A\cup B)^c &= A^c\cap B^c\\ (A\cap B)^c &= A^c\cup B^c\end{aligned}\right\}$, (De Morgan's Law)

**(ii)** Let $A$ and $B$ be subsets of a set $X$. Then, $A-B = A\cap B^c$.

**(iii)** $A^c - B^c = B - A$.

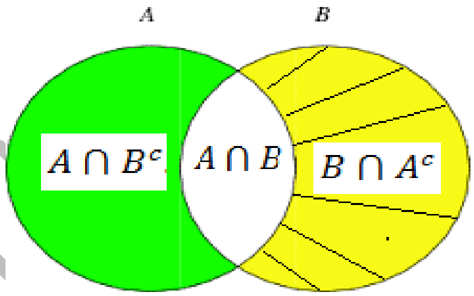
**Proof.**

**(i)** Let $x\in(A\cup B)^c$

| | |
|---|---|
| $\Leftrightarrow x\notin(A\cup B)$ | Def. of complement |
| $\Leftrightarrow \sim(x\in A\cup B)$ | Def. of $\notin$ |
| $\Leftrightarrow \sim(x\in A\vee x\in B)$ | Def. of A ∪ B |
| $\Leftrightarrow \sim(x\in A)\wedge\sim(x\in B)$ | De Morgan's Law |
| $\Leftrightarrow x\notin A\wedge x\notin B$ | Def of $\notin$ |
| $\Leftrightarrow x\in A^c\wedge x\in B^c$ | Def. of complement |
| $\Leftrightarrow x\in A^c\cap B^c$ | Def. of ∩ |

Hence $(A\cup B)^c = A^c\cap B^c$.

| | |
|---|---|
| **(ii)** $A-B=\{x\in X \mid x\in A \text{ and } x\notin B\}$ | |
| $=\{x\in X \mid x\in A \text{ and } x\in B^c\}$ | Def. of complement of $B^c$ |
| $= A\cap B^c$ | Def. of complement intersection |



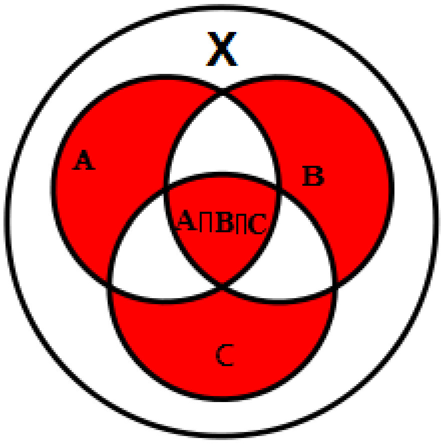
**(iii) Exercise.**

**Definition 2.2.18.** Let $A$ and $B$ be subsets of a set $X$. The set

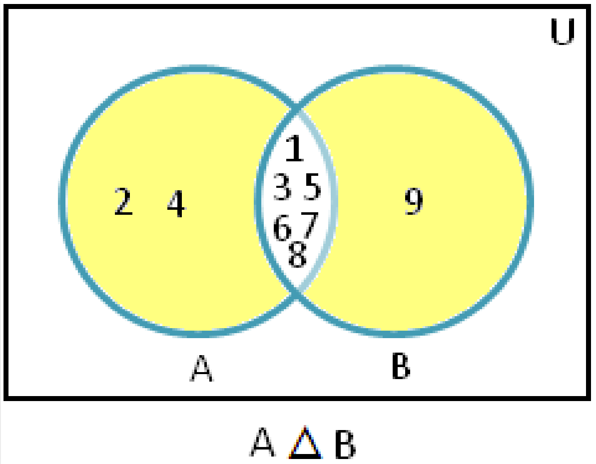
$$A\triangle B=(A-B)\cup(B-A)$$

is called the **symmetric difference.**

Sometimes the symbol $A\oplus B$ is used for symmetric difference.



**Example 2.2.19.** Let $A = \{1,2,3,4,5,6,7,8\}$ and $B = \{1,3,5,6,7,8,9\}$ are subsets of $U = \{1,2,3,4,5,6,7,8,9,10\}$.
$A - B = \{2,4\}$
$B - A = \{9\}$
$A \triangle B = (A - B) \cup (B - A) = \{2,4,9\}$.



**Theorem 2.2.20.** Let $A$, $B$ and $C$ are subsets of $X$. Then

**(i)** $A \triangle \emptyset = A$.
**(ii)** $A \triangle B = \emptyset \Leftrightarrow A = B$.
**(iii)** $A \triangle B = B \triangle A$.
**(iv)** $A \triangle A = \emptyset$.

**Proof. Exercise.**

**Theorem 2.2.21. (Properties of $\cup, \cap, -, \triangle$ and $P(X)$)**

**(i)** $A-(B\cap C)=(A-B)\cup(A-C)$ De Morgan's Low on $-$
$A-(B\cup C)=(A-B)\cap(A-C)$.

**(ii)** $A-(A\cap B)=(A-B)=(A\cup B)-B$,
$A-(A\cup B)=\emptyset$.

**(iii)** $(A\cap B)-C=(A-C)\cap(B-C)$
$(A\cup B)-C=(A-C)\cup(B-C)$.

**(iv)** $(A-B)\cap(C-D)=(C-B)\cap(A-D)$.

**(v)** If $A\subseteq B$, then $P(A)\subseteq P(B)$.

**(vi)** $P(A\cap B)=P(A)\cap P(B)$.

**(vii)** $P(A)\cup P(B)\subseteq P(A\cup B)$. The converse is not true.

**(viii)** $A=B \Leftrightarrow P(A)=P(B)$.

**(ix)** $A\cap B=\emptyset \Leftrightarrow P(A)\cap P(B)=\emptyset$.

**(x)** $A\triangle B=(A\cup B)-(A\cap B)$.

**(xi)** $A\triangle(B\triangle C)=(A\triangle B)\triangle C$. Associative Law of $\triangle$

**(xii)** $A\triangle C=B\triangle C \implies A=B$.

**(xiii)** If $A\subseteq B$ and $C=B-A$, then $A=B-C$.

**(xiv)** $A\cap(B-C)=(A\cap B)-(A\cap C)$.

**(xv)** $(A-B)\cap(C-D)=(A\cap C)-(B\cup D)$.

**(xvi)** $A\cap(B\triangle C)=(A\cap B)\triangle(A\cap C)$. Dist. of $\cap$ on $\triangle$

**Proof.**

**(i)** $A-(B\cap C)=A\cap(B\cap C)^c$ Theorem 2.2.17(ii)
$=A\cap(B^c\cup C^c)$ De Morgan's Law
$=(A\cap B^c)\cup(A\cap C^c)$ Dist. Law
$=(A-B)\cup(A-C)$. Theorem 2.2.17(ii)

**(vi)** Let $H\in P(A)\cap P(B)$
$\Leftrightarrow H\in P(A)\ \wedge\ H\in P(B)$ Def. $\cap$
$\Leftrightarrow H\subseteq A\ \wedge\ H\subseteq B$ Def. of power set
$\Leftrightarrow H\subseteq(A\cap B)$ Def. $\cap$
$\Leftrightarrow H\in P(A\cap B)$ Def. of power set

**(x)** $x\in A\triangle B \Leftrightarrow x\in(A-B)\cup(B-A)$ Def. of $\triangle$
$\Leftrightarrow x\in(A-B)\vee(B-A)$ Def. $\cup$
$\Leftrightarrow x\in A\ \wedge\ x\notin B\ )\ \vee\ (x\in B\ \wedge\ x\notin A)$ Def. of difference
$\Leftrightarrow (x\in A\ \vee\ x\in B)\ \wedge\ (x\notin B\ \vee\ x\in B)$ Dist. Law of
$\wedge$

|  |  |  |
|---|---|---|
|  | $(x \in A \ \vee \notin A) \quad \wedge \quad (x \notin B \ \vee \ x \notin A)$ |  |
| $\Leftrightarrow$ | $(x \in A \ \vee \ x \in B\ ) \wedge T$<br>$\wedge$<br>$T \quad \wedge \quad (x \notin B \ \vee \ x \notin A)$ | Tautology |
| $\Leftrightarrow$ | $x \in A \ \vee \ x \in B$<br>$\wedge$<br>$x \in B^c \ \vee \ x \in A^c$ | Identity Law of $\wedge$ |
| $\Leftrightarrow$ | $x \in (A \ \vee B)$<br>$\wedge$<br>$x \in (B^c \ \vee A^c)$ |  |
| $\Leftrightarrow$ | $x \in (A \ \cup B)$<br>$\cap$<br>$x \in (B^c \ \cup A^c) = (A \ \cap B)^c$ | Def. of $\cup$ and<br>De Morgan's Law |
| $\Leftrightarrow$ | $x \in (A \ \cup B) \cap (A \ \cap B)^c$ |  |
| $\Leftrightarrow$ | $x \in (A \ \cup B) - (A \ \cap B)$ | Theorem 2.2.17(ii) |

**Another proof of (x)**

|  |  |  |
|---|---|---|
| $A \triangle B$ | $= (A - B) \cup (B - A)$ | Def. of $\triangle$ |
|  | $= (A - (A \cap B)) \cup (B - (A \cap B))$ | Theorem 2.2.21(ii) |
|  | $= (A \cap (A \cap B)^c) \cup (B \cap (A \cap B)^c)$ | Theorem 2.2.17(ii) |
|  | $= ((A \cap B)^c \cap A) \cup \ ((A \cap B)^c \cap B)$ | Theorem 2.2.11(i) |
|  | $= (A \cap B)^c \cap (A \cup B)$ | Theorem 2.2.1(vii) |
|  | $= (A \cup B) \cap (A \cap B)^c$ | Inf. Theorem 2.2.11(i) |
|  | $= (A \ \cup B) - (A \ \cap B)$ | Inf. Theorem 2.2.17(ii) |

**Example 2.2.22:** Let $A = \{1,2\}, B = \{3\}$.
$P(A) = \{\emptyset, \{1\}, \{2\}, A\}$, $P(B) = \{\emptyset, B\}$.
$P(A \cup B) = P(\{1,2,3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.
$P(A) \cup P(B) \subset P(A \cup B)$.