# Lecture 3

## Advection

The way we describe the changes and motions occurring in the atmosphere is through
conservation laws:

- conservation of mass;
- conservation of momentum;
- conservation of energy.

These conservation laws apply to parcels of air in the same way that they apply to individual bodies or particles. However, in order to apply these conservation laws in an Eulerian frame, we must determine how forces (like gravity) and processes (like heating from condensation or radiation) change the values of quantities such as wind speed and direction or temperature at fixed points in space. In the Eulerian frame,
quantities do not change only as a result of forces and processes acting on the parcel;change at a given point can occur simply because one air parcel has moved on from that point in the frame and another air parcel has replaced it.

Consider an air parcel with a temperature T experiencing a steady southerly windas shown in Figure 2.10 in the left hand panel. Let us assume for now that there is no heating or cooling and hence in Lagrangian terms

$$DT/Dt = 0 \quad (2.17)$$

since there are no processes acting on the air parcels that can change the temperature.

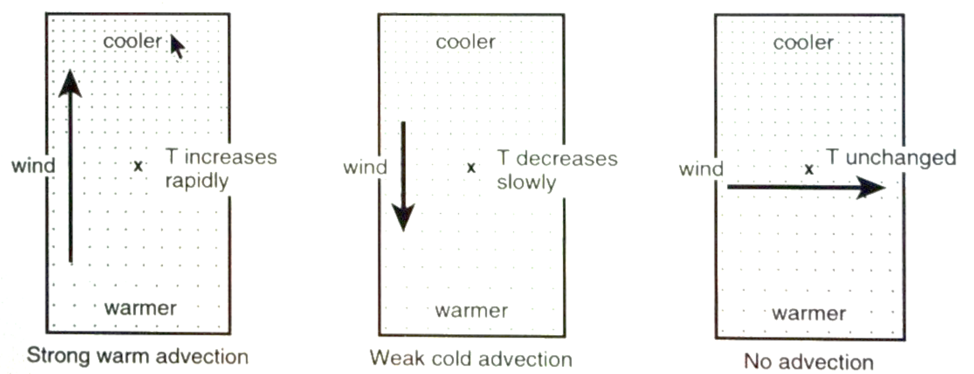


**Figure 2.10** Temperature change at a point in the presence of a strong southerly, a weak northerly, and a strong westerly wind

However, in Eulerian terms there will be a change in temperature irrespective of heating or cooling, since at this location, warmer air is replacing the cooler air that is flowing northward. This is called warm advection, and is what you experience when, in the Northern Hemisphere, you feel a warm southerly wind, for example.

If the wind were flowing in the other direction, as shown in Figure 2.10 in the center panel, then the temperature would decrease as cooler air parcels replaced warmer ones. This is known as cold advection.

The rate at which this advection occurs must depend on two things – the speed of the wind that is transporting the air parcels with differing temperatures, and the strength of the temperature gradient. Also important is the orientation of the temperature gradient with respect to the wind direction – if the wind is flowing along isotherms (lines of constant temperature) then no advection will take place (Figure 2.10, right hand panel).

Any quantity can be advected, although it is most easily understood in the case of temperature. Mass (in the form of density), momentum (in the form of the wind velocity vector), and water vapor can all be modified at a particular location by this process.

Now we turn to the mathematical formulation of advection. Deriving the advective rate of change of any quantity requires a relationship between:

• the rate of change of the quantity at a fixed point, the local or Eulerian derivative/  t, and the rate of change of the quantity following the motion, the substantial, material, or Lagrangian derivative D/Dt.

Consider again the temperature. For a given air parcel, the Lagrangian rate of change of temperature DT/Dt is only a function of time. However, at a particular point in the

Eulerian frame, the change in temperature is no longer just a function of time, but also of position in the fluid:

$$\frac{DT}{Dt} = \lim_{\delta t \to 0} \frac{\delta T}{\delta t}$$

$$\delta T = \left(\frac{\partial T}{\partial t}\right)\delta t + \left(\frac{\partial T}{\partial x}\right)\delta x + \left(\frac{\partial T}{\partial y}\right)\delta y + \left(\frac{\partial T}{\partial z}\right)\delta z$$

$$\Rightarrow \frac{DT}{Dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial x}\frac{Dx}{Dt} + \frac{\partial T}{\partial y}\frac{Dy}{Dt} + \frac{\partial T}{\partial z}\frac{Dz}{Dt}$$

$$= \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}$$

$$\frac{\partial T}{\partial t} = \frac{DT}{Dt} - \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) \qquad (2.18)$$

Example The air at a point 50 km north of a station is 3 C cooler than at the station (Figure 2.11). If the wind is blowing from the north-east at 20ms−1 and the air at the station is being heated by radiation at the rate of 1 Ch−1, what is the temperature change at the station?
The temperature change at the station is

$$\left.\frac{\partial T}{\partial t}\right|_{station} = \left.\frac{\partial T}{\partial t}\right|_{heating} + \left.\frac{\partial T}{\partial t}\right|_{advection}$$

$$= \frac{DT}{Dt} - \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right)$$

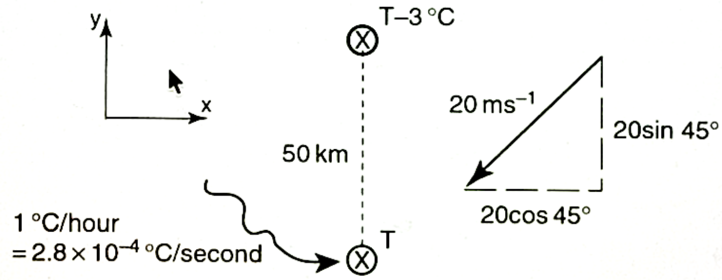


**Figure 2.11** Station with temperature *T* undergoing radiative heating and advective cooling

$$\frac{DT}{Dt} = \frac{1\,°C}{3600\,s} = 3 \times 10^{-4}\ °C\,s^{-1}$$

$$\vec{u} = -14\vec{i} - 14\vec{j}$$

$$\frac{\partial T}{\partial x} = 0 \qquad \frac{\partial T}{\partial y} = \frac{-3\,°C}{50\,km} = -6 \times 10^{-5}\ °C\,m^{-1} \qquad \frac{\partial T}{\partial z} = 0$$

$$\left.\frac{\partial T}{\partial t}\right|_{station} = 3 \times 10^{-4} - (-14 \times -6 \times 10^{-5})$$

$$= -5 \times 10^{-4}\ °C\,s^{-1}$$

Hence, in this case, the cooling by advection overwhelms the warming from the Sun, and the temperature at the station gets lower.