

1. prove $P = n_0 kT$

we have
$$n_0 = \frac{N_A n}{V} \dots\dots\dots(1)$$

where n_0 = number density (molecules m^{-3}), N_A = Avogadro's number ($N_A = 6.023 \times 10^{23}$ molecules mol^{-1}), n = number of moles (mole), v = volume (m^3).

From Equ. 6
$$PV = nR^*T \dots\dots\dots(2)$$

$$\therefore V = \frac{nR^*T}{P} \dots\dots\dots(3)$$

Substituting equation (1) into (3) we obtain:

$$n_0 = \frac{N_A n}{\frac{n R^*T}{P}}$$

$$n_0 = \frac{N_A P}{R^*T} \Rightarrow N_A P = n_0 R^*T \Rightarrow P = \frac{n_0 R^*T}{N_A}$$

we have $k = \frac{R^*}{N_A}$

So $P = n_0 kT$

2. prove

$$\frac{dp}{p} = -\frac{Mg}{R^*T} dz$$

Hydrostatic Equation $\frac{dp}{dz} = -\rho g \dots\dots\dots(1)$

Using the Ideal Gas Law, we can replace ρ and get the equation for dry air:

$$\frac{dp}{dz} = -g \frac{P}{R_d T} \Rightarrow \frac{dp}{p} = -\frac{g}{R_d T} dz \quad (2)$$

we have $R = \frac{R^*}{M}$ so for dry air $\Rightarrow R_d = \frac{R^*}{M_d} \dots\dots\dots(3)$

Substituting equation (3) into (2) we obtain:

$$\frac{dp}{p} = -\frac{Mg}{R^*T} dz$$

Exercise 2: Determine the apparent molecular weight of the Venusian atmosphere, assuming that it consists of 95% of CO₂ and 5% N₂ by volume. What is the gas constant for 1 kg of such an atmosphere? (Atomic weights of C, O, and N are 12, 16, and 14, respectively.)

Solution:

$$M_d = \frac{\sum_i m_i}{\sum_i \frac{m_i}{M_i}}$$
$$M_d = \frac{1}{\frac{1}{(12 + 32) \times 0.95 + (28) \times 0.05}} = 43.2 \text{ g}$$

$$R_d = 1000 \frac{R^*}{M_d} = 1000 \frac{8.3145}{43.2} = 192.46 \text{ JK}^{-1}\text{Kg}^{-1}$$