# Lecture 7

## Barotropic and Baroclinic Atmosphere, and Vertical Motion

### 7.1 Barotropic Atmosphere

A *barotropic* atmosphere is one in which the density depends only on the pressure, $\rho = \rho\ (p)$, so that the isobaric surfaces are also surfaces of constant density. For an ideal gas, the isobaric surfaces will also be isothermal if the atmosphere is barotropic. Thus, $\nabla_p T = 0$ in a barotropic atmosphere, and the thermal wind equation becomes equal to zero, which states that the geostrophic wind is independent of height in a barotropic atmosphere. Thus, barotropy provides a very strong constraint on the motions in a rotating fluid; the large-scale motion can depend only on horizontal position and time, not on height.

### 7.2 Baroclinic Atmosphere

An atmosphere, in which density depends on both the temperature and pressure, $\rho = \rho(p, T)$, is referred to as baroclinic atmosphere.

In a baroclinic atmosphere, the geostrophic wind has vertical shear, and this shear is related to horizontal temperature gradient by the thermal wind equation.

### 7.3 Vertical Motion

As mentioned previously, for synoptic-scale motions the vertical velocity component is typically of the order of a few centimeters per second. Routine meteorological soundings, however, only give the wind speed to an accuracy of about a meter per second. Thus, in general the vertical velocity is not measured directly but must be inferred from the fields that are measured directly.

Two commonly used methods for inferring the vertical motion field are the kinematic method, based on the equation of continuity, and the adiabatic method, based on the thermodynamic energy equation. Both methods are usually applied using the isobaric coordinate system so that $\omega(p)$ is inferred rather than $w(z)$.

These two measures of vertical motion can be related to each other with the aid of the hydrostatic approximation.

Expanding $dp/dt$ in the $(x, y, z)$ coordinate system yields

$$\omega \equiv \frac{dp}{dt} = \frac{\partial p}{\partial t} + \vec{V}_h . \nabla_h\, p + w\left(\frac{\partial p}{\partial z}\right) \qquad (7.1)$$

Now, for synoptic-scale motions, the horizontal velocity $\vec{V}_h$ is geostrophic to a first approximation. Therefore, we can write

$$\vec{V}_h = \vec{V}_g + \vec{V}_a$$

where $\vec{V}_a$ is the *ageostrophic* wind and $|\vec{V}_a| \ll |\vec{V}_g|$ . However, $\vec{V}_g = (\rho f)^{-1} k \times \nabla p$, so that

$$(\vec{V}_g + \vec{V}_a).\nabla_h\, p = \vec{V}_a.\nabla_h\, p$$

Where $\vec{V}_g.\nabla_h\, p = 0$ (the geostrophic wind is parallel to the pressure lines). Using this result plus the hydrostatic approximation, (7.1) may be rewritten as

$$\omega = \frac{\partial p}{\partial t} + \vec{V}_a.\nabla_h\, p - g\rho w \qquad (7.2)$$

Comparing the magnitudes of the three terms on the right in (7.2), we find that for synoptic-scale motions,

$$\frac{\partial p}{\partial t} \sim 10 \; hPa\, d^{-1}$$

$$\vec{V}_a.\nabla p \sim (1\, m\, s^{-1})(1\, Pa\, km^{-1}) \sim 1\, hPa\, d^{-1}$$

$$g\rho w \sim 100 \; hPa \; d^{-1}$$

Note: w is the vertical velocity with units of length per second. Travelling away from the surface means that w is positive. $\omega$ is the vertical velocity in pressure coordinates (so positive omega is negative w). Omega has units of pressure per time.

Thus, it is quite a good approximation to let

$$\omega = -g\rho w \qquad (7.3)$$

1. **Kinematic method**

One method of deducing the vertical velocity is based on integrating the continuity equation in the vertical. Integration of the continuity equation in the isobaric system

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p + \frac{\partial \omega}{\partial p} = 0$$

with respect to pressure from a reference level $p_s$ to any level $p$ yields,

$$\omega(p) = \omega(p_s) - \int_{p_s}^{p} \left(\frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y}\right)_p dp$$

$$\omega(p) = \omega(p_s) + (p_s - p)\left(\frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y}\right)_p \qquad (7.4)$$

Here the angle brackets denote a pressure-weighted vertical average:

$$\langle \quad \rangle \equiv (p - p_s)^{-1} \int_{p_s}^{p} ( \quad ) dp$$

With the aid of the (7.3), the averaged form of (7.4) can be rewritten as,

$$w(z) = \frac{\rho(z_s)\, w(z_s)}{\rho(z)} - \frac{p_s - p}{\rho(z)\, g} \left( \frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y} \right) \qquad (7.5)$$

where $z$ and $z_s$ are the heights of pressure levels $p$ and $p_s$, respectively.

Application of (7.5) to infer the vertical velocity field requires knowledge of the horizontal divergence. In order to determine the horizontal divergence, the partial derivatives $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ are generally estimated from the fields of $u$ and $v$ by using *finite difference approximations*.

From (7.4) we get the following conclusion:

**Conclusion**: **the vertical motion at a given pressure level is directly related to the integrated divergence above that level.**

Now, Also from (7.4) assuming a mean divergence from level $p_s$ to the top of the atmosphere $p = 0,\ w(p) = 0$.

$$\omega(p_s) = -p_s \left( \frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y} \right)_p$$

Possibilities:

1. If the mean divergence aloft $\left( \frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y} \right)_p > 0$ (*low pressure case*)

   and $p_s > 0 \Rightarrow \omega(p_s) < 0 \Longrightarrow rising\ motion$, (see Fig. 7.1)

2. If the mean convergence aloft $\left( \frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y} \right)_p < 0$ (*high pressure case*)

   and $p_s > 0 \Rightarrow w(p) > 0 \Longrightarrow subsidence\ motion\ (sinking)$, (see Fig. 7.2)

The location at which the divergence change its signs is known as the Level of Non-Divergence (LND). A maximum or minimum of w is found at LND, see Fig. 7.3.

Divergence at upper level is accompanied by convergence at lower level (Ascending motion).

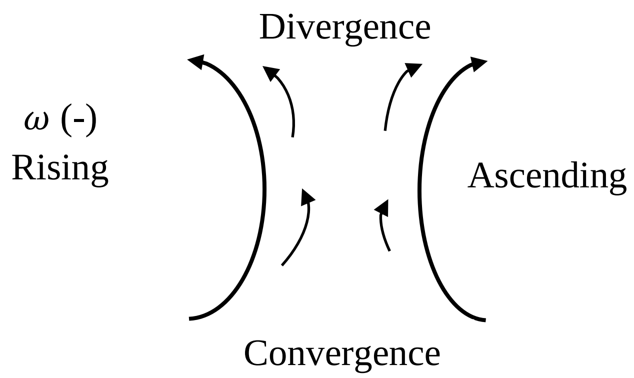


Fig. 7.1

Convergence at upper level is accompanied by divergence at lower level (Descending motion).

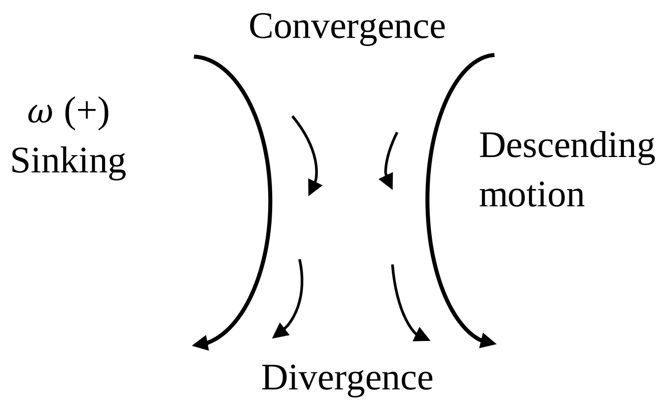


Fig. 7.2

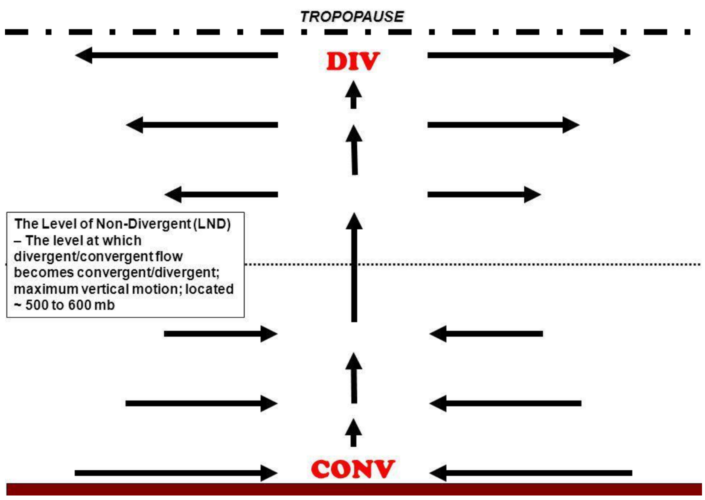


Fig. 7.3

## 2. The Adiabatic Method

A second method for inferring vertical velocity, which is not so sensitive to errors in the measured horizontal velocities, is based on the thermodynamics energy equation. The first law of thermodynamics is,

$$\frac{dH}{dt} = c_p \frac{dT}{dt} - \alpha \frac{dp}{dt}$$

$$J = c_p \left( \frac{\partial T}{\partial t} - u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial p} \right) - \alpha \frac{dp}{dt}$$

Using isobaric system

$$\frac{dp}{dt} = \omega$$

$$J = c_p \left( \frac{\partial T}{\partial t} - u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial p} \right) - \alpha\, \omega$$

$$\frac{J}{c_p} = \left( \frac{\partial T}{\partial t} - u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial p} \right) - \frac{\alpha}{c_p}\, \omega$$

Stability $S_p = \frac{R\,T}{c_p\,p}$ , so

$$\frac{J}{c_p} = \left( \frac{\partial T}{\partial t} - u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial p} \right) - S_p\, \omega$$

If we have geostrophic balance and adiabatic heating so;

$$\omega = S_p^{-1} \left( \frac{\partial T}{\partial t} - u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right)$$

Where T is the temperature and $S_p$ is the static stability parameter.

**Homework:**

1. What is barotropic atmosphere?
2. What is baroclinic atmosphere?
3. What are the differences between the barotropic and baroclinic atmospheres?
4. Derive the relationship between the vertical velocity "w" and the vertical velocity in pressure coordinates "$\omega$", equation (7.3)
5. Derive and explain the kinematic method of estimating vertical wind.
6. Derive and explain the adiabatic method of estimating vertical wind.