Lecture 4

Balanced Motion Part 2

4.1 The Gradient Wind

The gradient wind is defined as the wind existing if the trajectory of a particle (or air parcel) is circular and we have a balance among the pressure gradient force, the Coriolis force and the centrifugal force.

A. Cyclonic flow (low pressure)

In this case, a Coriolis force and the centrifugal force act in the same direction. In order to have a balance, the pressure gradient force must act in the opposite direction and we have a low pressure in the center (see case a in Figure 4.1). If we take the effect of curvature into account, we have to expand the horizontal momentum equation to include the centrifugal term:

$$PGF = CF + CeF \qquad (4.1)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial n} = \mathrm{f}\,\mathrm{V_G} + \frac{\mathrm{V_G^2}}{\mathrm{R}} \qquad (4.2)$$

and by using geostrophic balance $f\,V_g = -\frac{1}{\rho}\frac{\partial p}{\partial n}$, we substitute the left side in (4.2) by $\mathrm{f}\,\mathrm{V_g}$:

$$\mathrm{f}\,\mathrm{V_g} = \mathrm{f}\,\mathrm{V_G} + \frac{\mathrm{V_G^2}}{\mathrm{R}} \qquad (4.3)$$

Here $\mathrm{V_g}$ is the geostrophic wind, $\mathrm{V_G}$ is the gradient wind, and R is the radius of curvature.

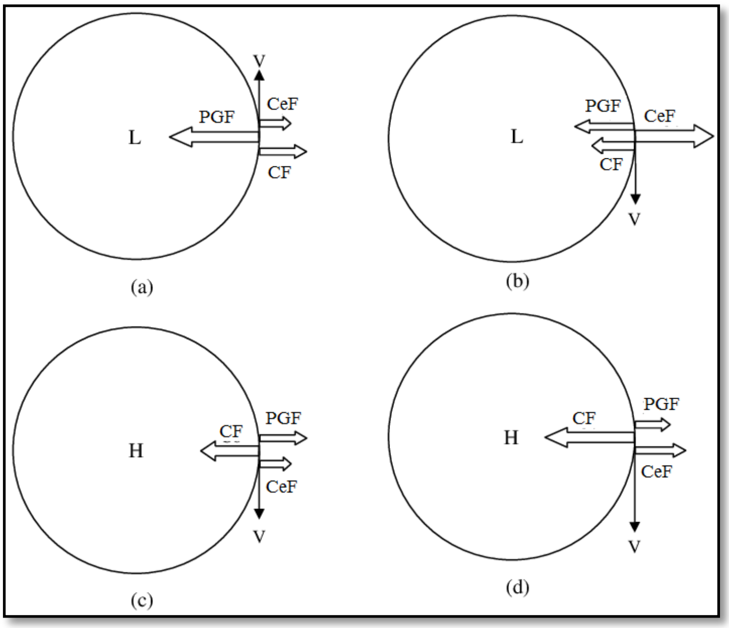


Fig. 4.1 Four balances for the four types of gradient flow.

The gradient wind speed is obtained by solving equation (4.3) for V_G to yield:

$$f\,V_g = f\,V_G + \frac{V_G^2}{R}$$

Dividing by V_G^2,

$$f\frac{V_g}{V_G^2} = \frac{f}{V_G} + \frac{1}{R}$$

$$fV_g\left(\frac{1}{V_G}\right)^2 - f\left(\frac{1}{V_G}\right) - \frac{1}{R} = 0$$

By using quadratic formula to solve,

$$x = \frac{-b \mp \sqrt{b^2 - 4\,a\,c}}{2\,a}$$

we get,

$$a = f\,V_g \qquad b = -f \qquad c = -\frac{1}{R} \qquad x = \frac{1}{V_G}$$

$$\frac{1}{V_G} = \frac{f \mp \sqrt{f^2 + 4\,\frac{f\,V_g}{R}}}{2\,f\,V_g}$$

Dividing the numerator and the denominator of the right side on $(2\,f)$ we get,

$$\frac{1}{V_G} = \frac{\frac{1}{2} \mp \sqrt{\frac{1}{4} + \frac{V_g}{R\,f}}}{V_g}$$

$$\therefore \qquad V_G = \frac{V_g}{\frac{1}{2} \mp \sqrt{\frac{1}{4} + \frac{V_g}{R\,f}}}$$

This equation tells us that $V_G < V_g$ in all cases because the denominator is larger than one. The difference between V_G & V_g becomes larger at smaller R, and at smaller latitude angle. To illustrate this difference we consider:

At $V_g = 10\frac{m}{s}$ $\quad and\ \ latitude = 45^o$

$if \quad R = 1000\,km$, we find $V_G = 9.18\,m/s$ and the difference between V_G & V_g is small.

When R becomes much smaller the difference between V_G & V_g will be large (for example at $R = 10\ km,\ \ V_G = 2.73\,m/s$) .

If we assume that $latitude = 45^o\ and\ \ V_g = 10\frac{m}{s}$ we may calculate the value of R necessary to make $V_G = \frac{1}{2}V_g$, we find from the equation that the radius of $R = 50\ km.$

See Table (4.1) for more details.

Table (4.1) The gradient wind speed at latitude 45° and $V_g = 10\ m/s$ at different R values for law pressure

No.	*R (m)*	*denominator only at (+) case*	*square root only*	*V_G at (+) case*	*V_G at (-) case*	*V_G at (+) case at latitude 30°*
1	1000	10.36003100	9.860031	0.965	-106.84	0.818250086
2	10000	3.653889842	3.153890	2.737	-3768.05	2.360273054
3	50000	1.979663552	1.479664	5.051	-51037.93	4.484402714
4	100000	1.604401247	1.104401	6.233	-165453.00	5.639112600
5	500000	1.166288543	0.666289	8.574	-3006821.70	8.169486737
6	**1000000**	**1.089041774**	**0.589042**	**9.182**	**-11230683.72**	**8.911043629**
7	2000000	1.046337904	0.546338	9.557	-43161209.60	9.394800613
8	4000000	1.023681729	0.523682	9.769	-168906589.24	9.678827156

B. Anticyclonic flow (high pressure)

In this case, a pressure gradient force and the centrifugal force are in the same direction. In order to have a balance the Coriolis force must act in the opposite direction, we have a high pressure in the center (case c and d in Fig. 4.1).

$$PGF + Ce\ F - CF = 0$$

$$f\ V_g + \frac{V_G^2}{R} - f\ V_G = 0$$

In the same previous manner,

$$\therefore \qquad V_G = \frac{V_g}{\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{V_g}{R\,f}}}$$

We see that $V_G > V_g$ in all cases.

In the special case where $\frac{V_g}{R\,f} = \frac{1}{4}$, $V_G = 2V_g$, the maximum wind in the anticyclonic case is therefore twice the geostrophic wind and hence if we assume that, $f = 10^{-4}\ s^{-1}$ and $V_g = 10\,m/s,$ the radius of curvature is equal to about 400 km, which is quite small.

Table (4.2) The gradient wind speed at latitude 45° and $V_g = 10\ m/s$ at different R values for high pressure

No.	*R (m)*	V_G
1	1000	-
2	10000	-
3	50000	-
4	100000	-
5	387881	19.98738189
6	500000	13.57277449
7	1000000	11.22094898
8	2000000	10.53847271
9	4000000	10.25494409

4.2 The Cyclostrophic Flow

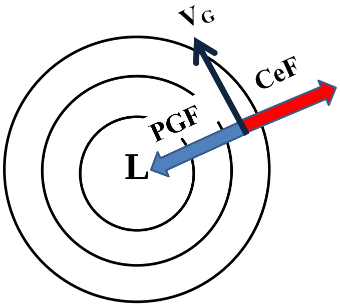
Cyclostrophic balance occurs when the pressure gradient force and centrifugal force are equal and in opposite direction. This is the situation near the equator

$$GF = Ce\ F$$

$$f\ V_g = \frac{V_G^2}{R}$$

$$V_G^2 = f\ V_g R$$

$$\therefore \quad V_G = \sqrt{f\ V_g R}$$



4.3 The Inertial Flow

In inertial flow, there is no pressure gradient force, there are two forces only, Coriolis and centrifugal that may balance each other.

$$CF = Ce\ F$$

$$f\ V_G = \frac{V_G^2}{R}$$

$$V_G = Rf$$