Lecture 3

Balanced Motion Part 1

3.1 Introduction

A balance motion is the one where the forces acting upon the particle at any time add up to zero.

$$\sum \vec{f_i} = 0$$

In most meteorological applications, we assume the apparent forces such Coriolis and centrifugal force as real forces. The approximate balance between some forces in the horizontal dimension give rise some important types of wind.

1. The Geostrophic Wind
   It is defined as the wind that would exist in the atmosphere if the motion were hydrostatic and horizontal without acceleration and friction.
   From the component of horizontal motion in momentum equations:

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu$$

On the assumption that the motion is without acceleration, the equations of motion becomes:

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv_g$$

$$v_g = \frac{1}{\rho f}\frac{\partial p}{\partial x} \qquad (3.1)$$

This is the geostrophic wind is in x-direction

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu_g$$

$$u_g = -\frac{1}{\rho f}\frac{\partial p}{\partial y} \qquad (3.2)$$

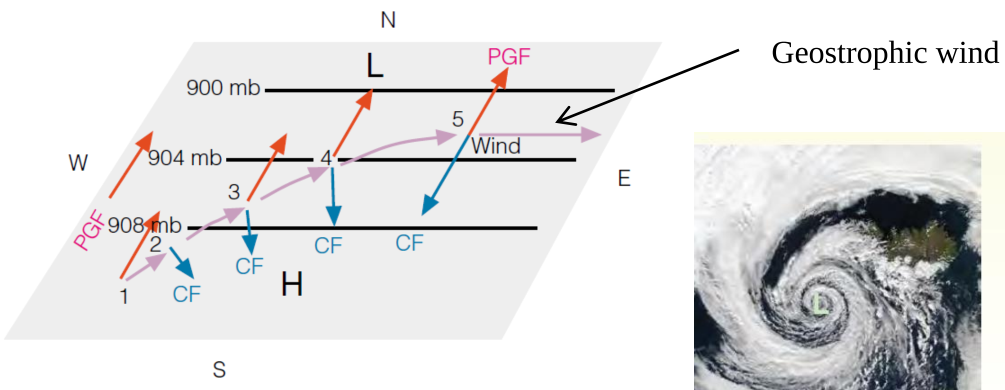
This is the geostrophic wind in y-direction.

The geostrophic wind vector can be written as:

$$\vec{V}_g = \frac{1}{\rho f}\, k \times \nabla p \qquad (3.3) \qquad (Note: each\ of\ k\ \&\ \nabla p\ are\ vectors)$$

where $f = 2\,\Omega\,\sin\phi$

The geostrophic wind relationship indicates that the wind blows in a direction normal to the horizontal pressure gradient (along the isobars), with lower pressure to the left in the northern hemisphere (see Fig. 3.1).



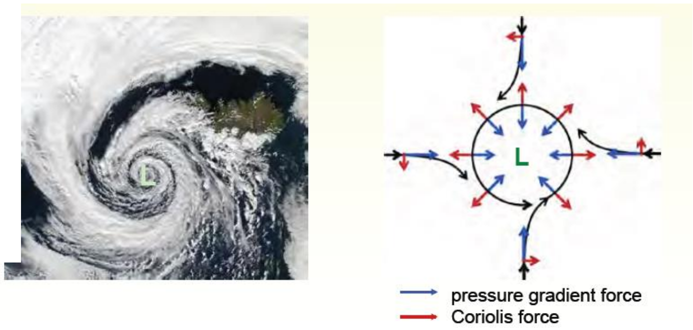


Fig. 3.1 Geostrophic wind

The greater pressure gradient is the greater geostrophic wind.

Hence, in N. H., the geostrophic wind is the wind that represents the balance between two equal and opposite forces, Pressure gradient force (PGF) and Coriolis force (CF). Figure 3.1 shows the relationship between the pressure gradient and the geostrophic wind speed. In the weather charts where the contours are drawn with equal increments, the more closely the contour lines the more speed the geostrophic wind.

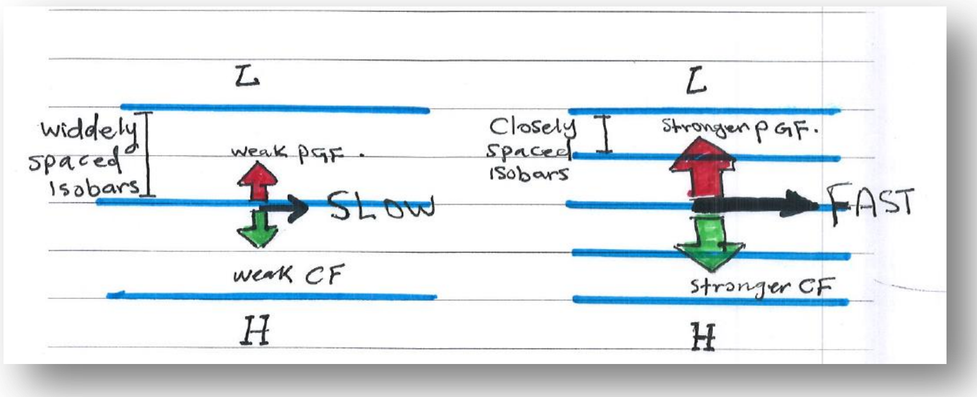


Fig. 3.2 The relationship between the pressure gradient and geostrophic wind speed

Question1: Show that the geostrophic wind can be given by:

$$\vec{V}_g = \frac{1}{\rho f}\, k \times \nabla p$$

<u>Sol.</u>

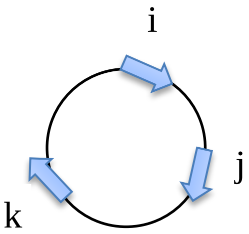
From eqns. (3.1) and (3.2):

$$(1) \quad v_g = \frac{1}{\rho f}\frac{\partial p}{\partial x} \qquad (2) \quad u_g = -\frac{1}{\rho f}\frac{\partial p}{\partial y}$$

By summing up the above two equations,

$$v_g j + u_g i = \frac{1}{\rho f}\frac{\partial p}{\partial x} j - \frac{1}{\rho f}\frac{\partial p}{\partial y} i$$

$$\vec{V}_h = \frac{1}{\rho f}\left(\frac{\partial p}{\partial x} j - \frac{\partial p}{\partial y} i\right)$$



now, by multiplying the two sides with the unit vector (k),

$$k \times \vec{V}_h = \frac{1}{\rho f}\left(\frac{\partial p}{\partial x} k \times j - \frac{\partial p}{\partial y} k \times i\right)$$

$$k \times \vec{V}_h = \frac{1}{\rho f}\left(-\frac{\partial p}{\partial x} i - \frac{\partial p}{\partial y} j\right)$$

$$k \times \vec{V}_h = \frac{-1}{\rho f}\left(\frac{\partial p}{\partial x} i + \frac{\partial p}{\partial y} j\right)$$

$$k \times \vec{V}_h = \frac{-1}{\rho f} \nabla_h p$$

Again, by multiplying the two sides with the unit vector (k),

$$k \times (k \times \vec{V}_h) = \frac{-1}{\rho f} \nabla_h p \times k$$

From the victorial relationship, $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C}$

$$k \times (k \times \vec{V}_h) = (k.\vec{V}_h)k - (k.k)\vec{V}_h$$

$$k \times (k \times \vec{V}_h) = 0 - \vec{V}_h$$

$$-\vec{V}_h = -\frac{1}{\rho f} k \times \nabla_h p$$

$$\therefore \vec{V}_h = \frac{1}{\rho f} k \times \nabla_h p$$

Question 2: Calculate the geostrophic wind speed as a function of the distance between the isobars (drawn with an interval of 5 mb) on a map scale 10^{-7} for latitude 60^{o} and density 1.2 kg/m^3. Assume distance one is 1 cm and distance 2 is 2 cm.

Sol.

$$f = 2\,\Omega\,\sin\phi = 2\times 7.3\times 10^{-5}\times \sin 60 = 12.6\times 10^{-5}\,sec^{-1}$$

$$Scale = \frac{\delta n_{map}}{\delta n_{earth}} \quad \Rightarrow \quad \delta n_{earth} = \frac{\delta n_{map}}{Scale}$$

At distance one, $\delta n_{map} = 1\,cm$

$$\delta n_{earth} = \frac{1}{10^{-7}} = 10^{7}\,cm = 10^{7}\times 10^{-2} = 10^{5}\,m$$

At distance two, $\delta n_{map} = 2\,cm$

$$\delta n_{earth} = \frac{2}{10^{-7}} = 2\times 10^{5}\,m$$

$$V_{g1} = \frac{1}{1.2\times 12.6\times 10^{-5}}\times\frac{5\times 10^{2}}{10^{5}} = 33\,m/s$$

$$V_{g2} = \frac{1}{1.2\times 12.6\times 10^{-5}}\times\frac{5\times 10^{2}}{2\times 10^{5}} = 16.5\,m/s$$

Question 3: What is the distance between the isobars (drawn with an interval of 5 mb) on a map of scale 1:10^7 at the latitude 30^{o} if the geostrophic wind speed is 20 m/s (density = 1.2 kg/m^3).

Sol.

$$f = 2\,\Omega\,\sin\phi = 2\times 7.3\times 10^{-5}\times \sin 30 = 7.3\times 10^{-5}\,sec^{-1}$$

$$V_g = \frac{1}{\rho f}\,\frac{\partial p}{\partial n_{earth}}$$

$$\partial n_{earth} = \frac{1}{\rho f}\,\frac{\partial p}{V_g}$$

$$\partial n_{earth} = \frac{1}{1.2\times 7.3\times 10^{-5}}\times\frac{5\times 10^{2}}{20} = 0.028\times 10^{7} m$$

$$Scale = \frac{\delta n_{map}}{\delta n_{earth}} \quad \Rightarrow \delta n_{map} = Scale\times \delta n_{earth}$$

$\delta n_{map} = 10^{-7}\times 0.028\times 10^{7} = 0.028\,m$ = 2.8 cm

Question 4: what is the geostrophic wind speed in the pressure gradient: $\frac{4}{100}\frac{mb}{km}$ where the density is 1.2 kg/ m^3 , the latitude is 60^o?

$$V_g = \frac{1}{\rho f}\frac{\partial p}{\partial n}$$

$$f = 2\,\Omega\,\sin\phi = 2\ \times 7.3\ \times 10^{-5} \times \sin 60 = 12\ \times 10^{-5}\ sec^{-1}$$

Case 1:

$$\frac{\partial p}{\partial n} = \frac{4}{100}\frac{mb}{km} = \frac{4\times 10^{2} N\,m^{-3}}{100\ \times 10^{3}\ m} = 4 \times 10^{-3} N\,m^{-3}$$

$$V_g = \frac{1}{1.2\ \times 12\ \times 10^{-5}}\ \times 4 \times 10^{-3}\ m\,s^{-1}$$

$$V_g = 26\,m\,s^{-1}$$

Case 2:

$$\frac{\partial p}{\partial n} = \frac{4}{200}\frac{mb}{km} = \frac{4\times 10^{2} N\,m^{-3}}{200\ \times 10^{3}\ m} = 2 \times 10^{-3} N\,m^{-3}$$

$$V_g = \frac{1}{1.2\ \times 12\ \times 10^{-5}}\ \times 2 \times 10^{-3}\ m\,s^{-1}$$

$$V_g \approx 13.8\,m\,s^{-1}$$

Discuss the results!