8 WORKED ANSWERS TO PROBLEM CLASSES

8.1 Complex Numbers

8.1.1 Complex Numbers Problem Class I

1. Solve the following quadratic equations for ‘x’ :-

(i) $x^2 + 2x + 5 = 0$

Use the quadratic formula :-

For an equation of the form $Ax^2 + Bx + C = 0$

$$x = \frac{-B \pm \sqrt{B^2 - (4AC)}}{2A}$$

Therefore, for $x^2 + 2x + 5 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - (4 \times 1 \times 5)}}{2 \times 1}$$

$$= \frac{-2 \pm \sqrt{4 - (20)}}{2}$$

$$= \frac{-2 \pm \sqrt{-16}}{2}$$

$$= \frac{-2 \pm \sqrt{-1 \times 16}}{2} = \frac{-2 \pm (4)\sqrt{-1}}{2}$$

By determining the square root of 16 and bringing it outside the square root term.

Now $\sqrt{-1}$ is denoted by j (this is what we defined !)

Therefore, $\frac{-2 \pm (4)\sqrt{-1}}{2} = \frac{-2 \pm j4}{2}$

So the two answers for ‘x’ are

$\frac{-2 + j4}{2} = -1 + j2$ By dividing every term by a factor of 2

or

$\frac{-2 - j4}{2} = -1 - j2$

(ii) $8x^2 + 6x + 2 = 0$

$$x = \frac{-6 \pm \sqrt{-28}}{16}$$

Therefore, $x = \dfrac{-6 - j\sqrt{28}}{16}$ or $\dfrac{-6 + j\sqrt{28}}{16}$

(iii) $2x^2 + 4x + 4 = 0$

$$x = \frac{-4 \pm \sqrt{-16}}{4}$$

Therefore, -1 + j or -1 - j

(iv) $10x^2 + 2x + 5 = 0$

$$x = \frac{-2 \pm \sqrt{-196}}{20}$$

Therefore, $x = \dfrac{-2 - j14}{20}$ or $x = \dfrac{-2 + j14}{20}$

(v) $x^2 + 2x + 17 = 0$

$$x = \frac{-2 \pm \sqrt{-64}}{2}$$

Therefore, x = -1 + j4 or - 1 - j4

2. Simple arithmetic of complex numbers. Calculate the following :-

(i) (3 + j6) + (9 + j2)

Add together like terms :-

Therefore, (3 + j6) + (9 + j2) = (3 + 9) + (j6 + j2) = 12 + j8

(ii) (-7 + j2) + (3 - j7) = - 4 - j5

(iii) (1 - j8) + (3 - j2) = 4 - j10

(iv) (8 - j2) - (6 + j7)

Subtract like terms :-
Therefore, (8 - j2) - (6 + j7) = (8 - 6) + (-j2 - j7) = 2 - j9
(v) (-3 - j9) - (25 + j15) = - 28 - j24
(vi) (2.3 + j7.8) - (6 - j2.4) = - 3.7 + j10.2

(vii) $(3+\sqrt{-16}) + (2+\sqrt{-9}) = (3 + j4) + (2 + j3)$ as $\sqrt{-9} = 3\sqrt{-1} = j3$

and $\sqrt{-16} = 4\sqrt{-1} = j4$

Now $(3 + j4) + (2 + j3) = 5 + j7$

(viii) $(4 - j2) - (4 - j3) = j$

(ix) $(j^2 + j6) - (4j^2 + j3) = (-1 + j6) - (-4 + j3)$

$= 3 + j3 \qquad (j^2 = -1)$

(x) $(7 + j2) - (7 + j2) = 0$

3. Multiplication of complex numbers. Calculate the following :-

(i) $(2 + j3)(1 + j6)$

Multiply out as if dealing with an algebraic expression, as follows :-

$(2 + j3)(1 + j6) = (2 \times 1) + (2 \times j6) + (j3 \times 1) + (j3 \times j6)$

$= 2 + j12 + j3 + j^2 18$

Now $j^2 = -1$

Therefore, $2 + j12 + j3 + j^2 18$ becomes $2 + j12 + j3 + -18 = -16 + j15$

(ii) $(-3 - j2)(4 + j5) = -2 - 23j$

(iii) $(7 + j8)(1 + j5) = -33 + j43$

(iv) $(3 + j1)(7 + j8) = 13 + j31$

(v) $(3+j2)^2 = (3 + j2)(3 + j2)$

$= 5 + j12$

(vi) $(3+j2)^3 = (3+j2)^2(3 + j2)$

$= (5 + j12)(3 + j2)$ As the answer to $(3+j2)^2$ was determined in the last question to be $(5 + j12)$

$= -9 + j46$

(vii) multiply $(3 - j5)$ by a complex number to give a real answer.

If (3 - j5) is multiplied by its conjugate a real number will be obtained. The conjugate of (3 - j5) is (3 + j5). The sign between the real and imaginary elements of the complex number is the only difference.

$(3 - j5)(3 + j5) = 9 + j15 - j15 - j^2 25$

$= 9 + 25 = 34$ (As $j^2 = -1$)

(viii) multiply (-4 + j7) by its conjugate.

The conjugate is (-4 - j7)

Therefore, (-4 + j7) (-4 - j7) = 65

(ix) $j^4 = \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1}$

Now $\sqrt{-1} \times \sqrt{-1} = -1$

Therefore $j^4 = -1 \times -1 = 1$

(x) $j^3 = \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} = -1 \times \sqrt{-1} = -j$

4. Division of complex numbers. Calculate :-

(i) $\frac{6 - j12}{3}$ Simply divide through by the denominator (bottom number)

i.e. $\frac{6}{3} - j\frac{12}{3} = 2 - j4$

(ii) 2 - j4

(iii) $\frac{3 - j4}{7 + j2}$ Multiply top and bottom by the complex conjugate of the bottom. The conjugate is the complex number with only the sign in the middle changed.

i.e. for 7 + j2 the conjugate is 7 - j2

So for $\frac{3 - j4}{7 + j2}$ multiply top and bottom by 7 - j2

$$\frac{(3 - j4)}{(7 + j2)}\frac{(7 - j2)}{(7 - j2)} = \frac{21 - j6 - j28 + j^2 8}{49 - j14 + j14 - j^2 4}$$

(remember the brackets are multiplied out by handling the expression as if it were an algebraic expression)

$$= \frac{21 - j34 - 8}{53} = \frac{13 - j34}{53}$$

$$= \frac{13}{53} - j\frac{34}{53}$$

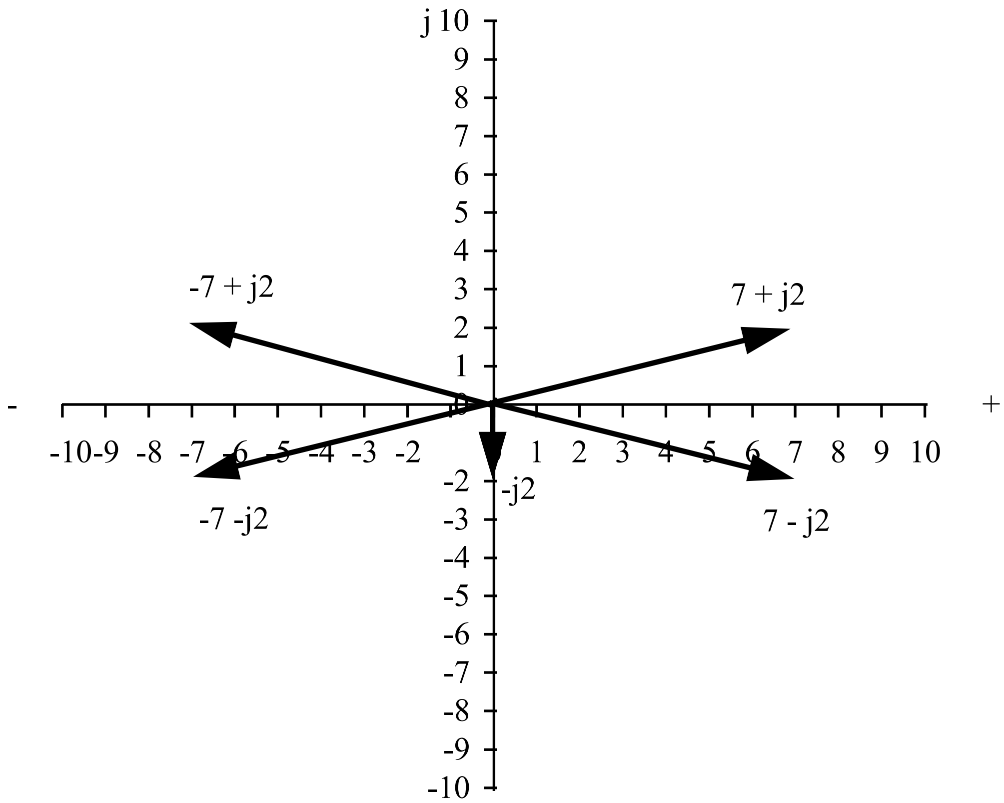
(iv) $\frac{6 + j8}{2 + j7} = \frac{68}{53} - j\frac{26}{53}$

(v) $\frac{-3 - j2}{4 - j5} = -\frac{2}{41} - j\frac{23}{41}$ (vi) $\frac{4 + j3}{4 + j3} = 1$!

5. Represent the following on an Argand Diagram :-

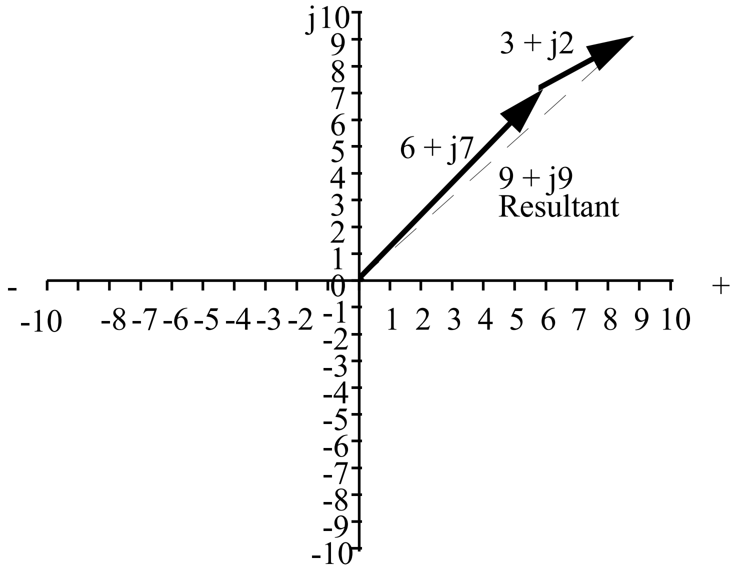
(i) 7 + j2 (ii) 7 - j2 (iii) -7 + j2 (iv) -7 - j2

(v) -j2



6. Using an Argand Diagram calculate the following :-

(i) (6 + j7) + (3 + j2)

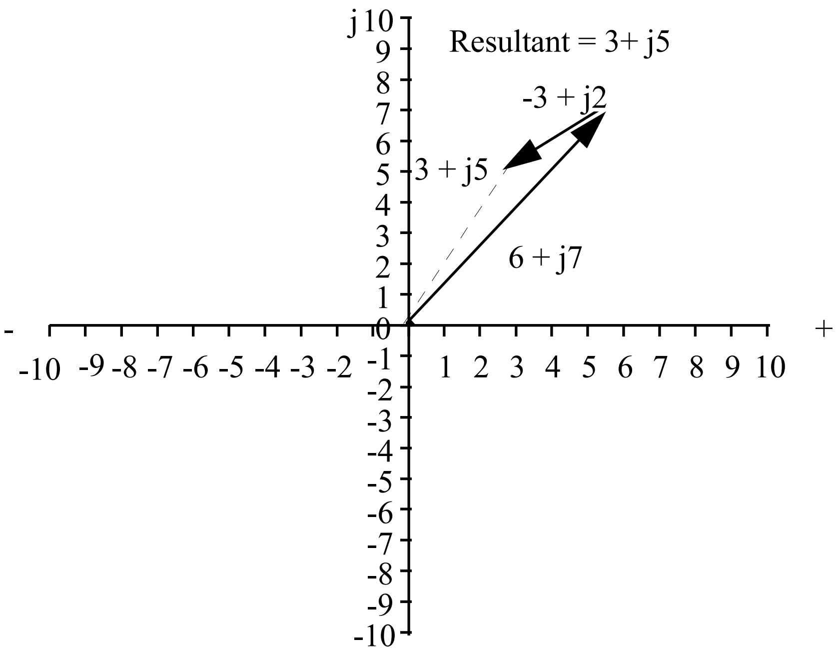


(ii) (4 + j3) + (5 + j2), resultant = 9 + j5

(iii) (-6 + j2) + (3 + j), resultant = -3 + j3

(iv) (-8 - j) + (3 - j), resultant = -5 - j2

(v) (6 + j7) - (3 + j2)



(vi) $(4 + j3) - (5 + j2)$, resultant $= -1 + j$