**9. Fundamental Theorems of Homomorphism**

**The First Fundamental Theorem of Isomorphism:**

**Theorem(9-1):**Let be an onto, homomorphism, then

.

**Proof:** is an onto

is a group.

Define

Let

is a map.

Let

is an one to one.

is onto.

is a homomorphism, hence is an isomorphism

**Example(9-2):** Let

, show that by two ways.

1. Since and are cyclic groups
2. By use the first theorem of isomorphism it is clear that is a homomorphism. Cod

is an onto

is a cyclic group is a cyclic

**Corollary(9-3):** Let be a group, then , where is a center of .

**Proof:** define

Let

is a map.

is a homomorphism.

is an onto.

By the first theorem of isomorphism

**The Second Theorem of Isomorphism:**

**Theorem(9-4):** Let be two subgroups of , then

1. is a subgroup of

**Proof:** since is a group.

And since is a group.

Define

is a map.

Thus, is an onto.

Since

Let

is a homo.

By the first theorem of isomorphism

Therefore,

**The Third Fundamental Theorem of Isomorphism:**

**Theorem(9-5):** Let be two normal subgroups of , then:

**Proof:** 1. Since are subgroups and

is a subgroup of

Let

Thus, .

**Proof:** 2. since is a group

Since is a group

is a subgroup of

Let

**Proof:** 3. is a group.

is a group.

Define

is a map.

is an onto.

is a homomorphism.

By the first theorem of isomorphism

ker

Therefore, .