**2. Some Properties of Groups**

**Theorem(2-1):** If a group, then the left and right cancellation laws hold in , that is:

1. .

**Proof:** 1. Suppose , then

.

(2) **(Homework).**

**Theorem(2-2):** In a group , there is exactly one element in such that .

**Proof:** Assume that has two identity elements and , this means for all , we have and

and .

**Theorem(2-3):** In a group , the inverse element of each element of is a unique.

**Proof:** Let and has two inverses and, such that

and

.

**Theorem(2-4):** If is a group, then

**Proof:** 1. Let

From 1 and 2, .

(2)

.

(3) since

.

**Theorem(2-5):** Let be a group, then

1. iff is an abelian group.
2. If , then is an abelian group.

**Proof:** i. let be a group and

To prove is an abelian group.

Let , to prove

let be an abelian group, to prove

.

(ii) let ,

to prove

.

**Remark(2-6):** The converse of above part is not true, for example let be an abelian group with .

**Theorem(2-7):** In a group , the equations and have a unique solutions.

**Proof:**

To show the solution is a unique

Let

.

The proof of (**Homework).**