**3. Certain Elementary Theorems on Groups.**

**Definition(3-1):** Let be a group, the integer powers of , is defined by:

1. (-times)

**Example(3-2):** In , we have

**Example(3-3):** In , we have

,

,

**Example(3-4):** In , we have

,

,

**Theorem(3-5):** Let be a group and , then:

1. (**Homework)**
2. is an abelian group.

**Proof:** (2) let

if

is a true.

Suppose that is a true with

We have to prove that is a true

is a true

By the principle of mathematical indication

is a true .

(3) if

Suppose that if is a true

We must prove is a true

is a true

By the principle of mathematical indication

is a true .

(4) if

is an abelian group

let be an abelian group and

If is a true

Suppose that is a true with

We must prove is a true

is a true .

**Definition(3-6):** (The order of a Group)

The number of elements of a group is called the order of and it is denoted by or . The group is called a finite if and an infinite group otherwise.

**Definition(3-7):** (The order of an element)

The order of an element , is the least positive integer such that where is the identity element of . We denoted to order by or . This means if .

**Example(3-8):** is an infinite group.

**Example(3-9):** The trivial group , , is the only group of order one.

**Example(3-10):** Find the order of and the order of their elements, where.

**Solution:** and ,

and .

**Exercises:**

* Find the order of and the order of their elements**.**
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* Let be a set. Define a binary operation on by the following table

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Find the order of and their elements