

ESCI 341 – Atmospheric Thermodynamics
Lesson 15 – Humidity
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PARTIAL PRESSURE

- In a mixture of gases, each gas species contributes to the total pressure.
 - The pressure exerted by a single gas species is known as the *partial pressure* for that species.
 - For a mixture of ideal gases, the partial pressure of any species can be found from the ideal gas law applied to that species only. For example, in air the partial pressures of O₂, N₂, and Ar would be

$$\begin{aligned}p_{O_2}V &= n_{O_2}RT; & p_{O_2} &= \rho_{O_2}R'_{O_2}T \\p_{N_2}V &= n_{N_2}RT; & p_{N_2} &= \rho_{N_2}R'_{N_2}T \\p_{Ar}V &= n_{Ar}RT; & p_{Ar} &= \rho_{Ar}R'_{Ar}T.\end{aligned}$$

- The densities used in the above equations are *partial densities*.
- The total pressure is equal to the sum of the partial pressures, and the total density is equal to the sum of the partial densities.
- The partial pressure of a species is proportional to the number of moles of the species.
- The partial pressure of a species can be found by multiplying the total pressure by the volume (or mole) fraction of the species. For example, the partial pressure of O₂ is found by multiplying atmospheric pressure by 0.21 (21%).

VAPOR PRESSURE

- If you have a substance in liquid form, some of the molecules will escape into the vapor phase. The partial pressure due to these vapor molecules is known as the *vapor pressure*.
- Since the vapor pressure due to water molecules is proportional to the number of water vapor molecules in the atmosphere, vapor pressure is one measure of humidity. We usually denote vapor pressure as *e*. (From now on we will use the term vapor pressure to mean the vapor pressure due to water).
- *Absolute humidity* is defined as the mass of water vapor per unit volume. It is merely the density of the water vapor, ρ_v

- Vapor pressure is related to absolute humidity via the ideal gas law,

$$e = \rho_v R_v T,$$

where R_v is the specific gas constant for water vapor (461 J·kg⁻¹·K⁻¹).

SATURATION VAPOR PRESSURE

- The vapor molecules are colliding with each other. Some may stick together briefly to form tiny water droplets. However, these tiny water droplets are also constantly breaking apart.
- If enough vapor molecules are present, there may be enough collisions to form a stable population of liquid water droplets. This is called *saturation*, and the vapor pressure at this point is called the *saturation vapor pressure* (e_s).
 - *Note: You often hear this expressed in terms of the air not being able to hold any more water vapor. This is really poor terminology, because it doesn't matter what other gasses are present. There could be only pure nitrogen, pure helium, or even nothing else present. The saturation vapor pressure would not change!*
- The saturation vapor pressure is a function of temperature, and is given by the Clausius-Clapeyron equation

$$e_s = e_0 \exp \left[\frac{L_v}{R_v} \left(\frac{1}{T_0} - \frac{1}{T} \right) \right],$$

where e_0 is the vapor pressure at some known temperature T_0 , and L is the latent heat of vaporization. We typically use $T_0 = 273\text{K}$, $e_0 = 611\text{ Pa}$, and $L_v = 2.5 \times 10^6\text{ J·kg}^{-1}$.

SPECIFIC HUMIDITY AND MIXING RATIO

- Vapor pressure and absolute humidity are not very convenient expressions for humidity (at least for meteorologists). We define some other measures of humidity.
- *Mixing ratio*: The mass of water vapor per mass of *dry* air. This would be dimensionless (if expressed as kg/kg, or g/g). However, it is more often expressed as grams of water vapor per kg of dry air, so the *units of mixing ratio are usually expressed as g/kg*.

- Mixing ratio can be related to vapor pressure via

$$r = \frac{\rho_v}{\rho_d} = \frac{\frac{e}{R_v T}}{\frac{p_d}{R_d T}} = \frac{R_d}{R_v} \frac{e}{p_d} = \varepsilon \frac{e}{p - e},$$

where $\varepsilon = R_d/R_v$.

- The saturation mixing ratio, r_s , is found by using e_s in the formula.
- **Specific humidity:** The mass of water vapor per mass of air.
 - Specific humidity is very close to mixing ratio, as shown

$$q = \frac{\rho_v}{\rho} = \frac{\rho_v}{\rho_d + \rho_v} = \frac{\frac{\rho_v}{\rho_d}}{1 + \frac{\rho_v}{\rho_d}} = \frac{r}{1 + r} \approx r,$$

since $r \ll 1$ (expressed as g/g or kg/kg).

- In meteorology, mixing ratio is used far more than specific humidity, and for most purposes the two can be considered as equivalent.

DEW POINT TEMPERATURE

- The saturation vapor pressure is a function of temperature, and decreases with decreasing temperature.
- If you cool moist air, eventually the saturation vapor pressure will equal the vapor pressure, and saturation will be reached. The temperature at which this occurs is called the *dew point*, or *dew point temperature*.
- *It is important to note that it is the saturation vapor pressure, and not the actual vapor pressure, that is changing as you cool the air!*
- The dew point temperature can be found from the Clausius-Clapeyron equation by using the actual vapor pressure instead of the saturation vapor pressure, and solving for T . This gives

$$T_d = \left[\frac{1}{T_0} - \frac{R_v}{L} \ln \left(\frac{e}{e_0} \right) \right]^{-1}.$$

RELATIVE HUMIDITY

- Specific humidity, mixing ratio, vapor pressure, and dew point remain unchanged if the air parcel is heated or cooled at constant pressure.
 - In other words, if I want to compare two air samples to find which has more water vapor, I can directly compare their mixing ratios, vapor pressures, specific humidities, or dew points. The one with the higher number will have more water vapor.
 - I can compare the samples without worrying about whether they are at the same temperature or not (however, vapor pressure will have to be compared at the same air pressure)!
 - This is not true of absolute humidity, which will change as the air parcel is heated or cooled.
- Relative humidity, as its name implies, is a relative measure of humidity. It is defined as the ratio of the vapor pressure to the saturation vapor pressure,

$$RH \equiv \frac{e}{e_s} \times 100\% .$$

- In terms of mixing ratio, relative humidity becomes

$$\frac{RH}{100\%} \equiv \frac{e}{e_x} = \frac{\frac{pr}{\varepsilon + r}}{\frac{pr_s}{\varepsilon + r_s}} = \frac{(\varepsilon + r_s) r}{(\varepsilon + r) r_s} \approx \frac{r}{r_s} .$$

- This means we can use mixing ratio rather than vapor pressure to find relative humidity.
- Relative humidity tells us how close an air parcel is to saturation.
 - It does not directly tell us how much water vapor is in the parcel!
- *A parcel with higher relative humidity may actually have less water vapor than another parcel with lower relative humidity!*

KEY DIFFERENCE BETWEEN THE MEASURES OF HUMIDITY

- There are two ways to change the relative humidity, or absolute humidity of an air parcel:
 - Add or subtract water vapor.

- Change the temperature.
- There is only one way to change mixing ratio, specific humidity, vapor pressure, or dew point (assuming pressure is constant):
 - Add or subtract water vapor.

HUMIDITY MEASUREMENT

- Meteorologists commonly measure humidity by measuring the *wet-bulb temperature*.
 - The wet-bulb temperature is the lowest temperature that can be achieved by evaporating water into the air parcel at constant pressure (the evaporation requires heat, which is supplied by the air parcel).
 - Wet-bulb temperature is measured using a *psychrometer*.
- The difference between the air temperature (*dry-bulb temperature*) and the wet-bulb temperature is called the *wet-bulb depression*.
 - The wet-bulb depression is a relative measure of the moisture content of the air. Dry air can be cooled much further by evaporation than moist air, so a larger wet-bulb depression means less humidity (for the same dry-bulb temperature).
- The dew-point temperature and relative humidity are found by using *psychrometric tables*, with dry-bulb temperature and wet-bulb depression as the independent variables.

VIRTUAL TEMPERATURE

- Moist air can be considered a mixture of two ideal gases (dry air and water vapor).
- The ideal gas law for moist air is

$$p = p_d + e = (\rho_d R_d + \rho_v R_v) T.$$

- We can manipulate this in the following manner

$$\begin{aligned} p &= \rho R_d T \left(\frac{\rho_d}{\rho} + \frac{\rho_v}{\rho} \frac{R_v}{R_d} \right) = \rho R_d T \left(\frac{\rho - \rho_v}{\rho} + \frac{\rho_v}{\rho} \frac{R_v}{R_d} \right) \\ &= \rho R_d T \left(1 - \frac{\rho_v}{\rho} + \frac{\rho_v}{\rho} \frac{R_v}{R_d} \right) = \rho R_d T (1 - q + q/\varepsilon) \end{aligned}$$

- If we define a new temperature, T_v such that

$$T_v = T(1 - q + q/\varepsilon)$$

then we can write the ideal gas law for moist air as

$$p = \rho R_d T_v.$$

- T_v is called the *virtual temperature*.
- For moist air we can use the ideal gas law for dry air, only using the virtual temperature in place of the actual temperature.
- Virtual temperature is always greater than or equal to the actual temperature.
 - The addition of water vapor causes the air to behave as though it is warmer. This makes sense, because moist air is lighter than dry air.
- Since $\varepsilon = 0.622$, we have

$$T_v = T(1 + 0.61 q). \quad \text{Virtual Temperature}$$

- Virtual temperature can also be written in terms of mixing ratio as

$$T_v = T \left(\frac{1 + r/\varepsilon}{1 + r} \right).$$

- Since mixing ratio and specific humidity are so close, we often write virtual temperature using mixing ratio as

$$T_v \cong T(1 + 0.61 r).$$

- In all of the preceding equations for virtual temperature we must use the absolute (Kelvin) temperature and the dimensionless form of mixing ratio or specific humidity.
 - However, there is an approximate formula for virtual temperature in Celsius that uses the dimensional (g/kg) form of mixing ratio or specific humidity. This formula is

$$T_{v[^\circ\text{C}]} \cong T_{[^\circ\text{C}]} + \frac{r_{[\text{g/kg}]}}{6}$$

and is often used in operational meteorology (if you are interested, you can find a derivation of this formula at [this link](#).)

EXERCISES

1. Show that the specific gas constant for water vapor, R_v , is $462 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$.
($R = 8.3145 \text{ J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$, and the molecular weight of water is 18.001 g/mol)
2. An air sample at standard sea level pressure, and with a volume of 1m^3 at 20°C , contains 7 grams of water vapor.
 - a. What is the vapor pressure?
 - b. What is the saturation vapor pressure?
 - c. What is the relative humidity?
 - d. What is the absolute humidity?
 - e. What is the mixing ratio?
 - f. What is the specific humidity?
 - g. What is the dew-point temperature?
3. The air parcel in question 2 is cooled (at constant pressure) to 10°C .
 - a. What is the vapor pressure?
 - b. What is the saturation vapor pressure?
 - c. What is the relative humidity?
 - d. What is the absolute humidity?
 - e. What is the mixing ratio?
 - f. What is the specific humidity?
 - g. What is the dew-point temperature?
 - h. Which humidity variables remained constant as a result of the cooling?