Example(3-8):

Are the following groups $(S_3,\circ)$ and $(G_s,\circ)$ have sylow p-subgroups.

Solution:

$(S_3,\circ)$, $O(S_3) = 6 = (2)(3)$,

$2 \setminus 6 \Longrightarrow \exists$ a subgroup H such that $o(H) = 2$ which is called sylow 2- subgroup.

Also, $3 \setminus 6 \Longrightarrow \exists$ a subgroup K such that $o(K) = 3$ which is called sylow 3- subgroup.

$(G_s,\circ)$, $o(G_s) = 2^3$ is 2- subgroup.

Every subgroup of G_s is 2- subgroup, $o(H) = 2^0$ or 2^1 or 2^2 or 2^3 .

Theorem(3-9): (Second Sylow Theorem)

The number of distinct sylow p-subgroups is $k = 1 + tp, t = 0,1, \ldots$ which is divide the order of G.

Example(3-10):

Find the distinct sylow p-subgroups of $(S_3,\circ)$.

Solution:

$o(S_3) = 6 = (2)(3)$,

$2 \setminus 6 \Longrightarrow \exists$ a subgroup H such that $o(H) = 2$.

The number of sylow 2-subgroups is $k_1 = 1 + 2t, t = 0,1, \ldots$ and $k_1 \setminus 6$

if $t = 0 \Longrightarrow k_1 = 1$ and $1 \setminus 6$

if $t = 1 \Longrightarrow k_1 = 3$ and $3 \setminus 6$

if $t = 2 \Longrightarrow k_1 = 5$ and $5 \nmid 6$

if $t = 3 \Longrightarrow k_1 = 7$ and $7 \nmid 6$

so, there are two sylow 2-subgroups.

$3 \setminus 6 \Longrightarrow \exists$ a subgroup K such that $o(K) = 3$.

The number of sylow 3-subgroups is $k_2 = 1 + 3t, t = 0,1, \ldots$ and $k_2 \setminus 6$

if $t = 0 \Longrightarrow k_2 = 1$ and $1 \setminus 6$

if $t = 1 \Longrightarrow k_2 = 4$ and $4 \nmid 6$

if $t = 2 \Longrightarrow k_2 = 7$ and $7 \nmid 6$

So, there is one sylow 3-subgroup.

Example(3-11):

Find the number of sylow p-subgroups of G such that $o(G) = 12$.

Solution: $o(G) = 12 = (3)(2^2)$

$3 \setminus 12 \Longrightarrow \exists$ a subgroup H such that $o(H) = 3$.

The number of sylow 3-subgroups is $k_1 = 1 + 3t, t = 0,1, \dots$ and $k_1 \setminus 12$

if $t = 0 \Longrightarrow k_1 = 1$ and $1 \setminus 12$

if $t = 1 \Longrightarrow k_1 = 4$ and $4 \setminus 12$

if $t = 2 \Longrightarrow k_1 = 7$ and $7 \nmid 12$

if $t = 3 \Longrightarrow k_1 = 10$ and $10 \nmid 12$

So, there are two sylow 3-subgroups of G.

The number of sylow 2-subgroups is $k_2 = 1 + 2t, t = 0,1, \dots$ and $k_2 \setminus 12$

if $t = 0 \Longrightarrow k_2 = 1$ and $1 \setminus 12$

if $t = 1 \Longrightarrow k_2 = 3$ and $3 \setminus 12$

if $t = 2 \Longrightarrow k_2 = 5$ and $5 \nmid 12$

if $t = 3 \Longrightarrow k_2 = 7$ and $7 \nmid 12$

So, there are two sylow 2-subgroups of G.