1. Sylow Theorem

Definition(3-1): (***Sylow p- Subgroup***)

Let $(G,*)$ be a finite group and p is a prime number, a subgroup $(H,*)$ of a group G is called *sylow p- subgroup* if

1. $(H,*)$ is a p- group,
2. $(H,*)$ is not contained in any other p- subgroup of G for the same prime number p.

Example(3-2);

Find sylow 2- subgroups and sylow 3- subgroup of the group $(Z_{24}, +_{24})$.

Solution: The proper subgroups of the group $(Z_{24}, +_{24})$ are

1. $(\langle 2\rangle, +_{24}) \Longrightarrow o(\langle 2\rangle) = 12 \neq P^k \Longrightarrow \langle 2\rangle$ is not p-subgroup.
2. $(\langle 3\rangle, +_{24}) \Longrightarrow o(\langle 3\rangle) = 8 = 2^3 \Longrightarrow \langle 3\rangle$ is a 2-subgroup.
3. $(\langle 4\rangle, +_{24}) \Longrightarrow o(\langle 4\rangle) = 6 \neq P^k \Longrightarrow \langle 4\rangle$ is not p-subgroup.

4. $(\langle 6\rangle, +_{24}) \Longrightarrow o(\langle 6\rangle) = 4 = 2^2 \Longrightarrow \langle 6\rangle$ is a 2-subgroup.
5. $(\langle 8\rangle, +_{24}) \Longrightarrow o(\langle 8\rangle) = 3 = 3^1 \Longrightarrow \langle 8\rangle$ is a 3-subgroup.
6. $(\langle 12\rangle, +_{24}) \Longrightarrow o(\langle 12\rangle) = 2 = 2^1 \Longrightarrow \langle 12\rangle$ is a 2-subgroup.

Theorem(3-3): (First Sylow Theorem)

Let $(G,*)$ be a finite group of order $\mathrm{p}^k q$, where p is a prime number is not dividing q, then G has sylow p- subgroup of order p^k.

Example(3-4):

Find sylow 2- subgroup of the group $(\mathrm{Z}_{12}, +_{12})$.

Solution: $o(\mathrm{Z}_{12}) = 12 = (4)(3) = (2^2)(3)$, and $2 \nmid 3$

$\Longrightarrow$ by first sylow theorem, the group $(\mathrm{Z}_{12}, +_{12})$ has sylow 2- subgroup of order 2^2.

$\Longrightarrow (\langle 3\rangle, +_{12})$ is a sylow 2- subgroup.

Example(3-5):

Find sylow 7- subgroup of the group $(Z_{42}, +_{42})$.

Solution: $o(Z_{42}) = 42 = (7)(6)$, and $7 \nmid 6$

$\Longrightarrow$ by first sylow theorem, the group $(Z_{42}, +_{42})$ has sylow 7- subgroup of order 7^1.

$\Longrightarrow$ $(\langle 6 \rangle, +_{42})$ is a sylow 7- subgroup.

Example(3-6):

Find sylow 3- subgroup of the group $(Z_{24}, +_{24})$.

Solution: $o(Z_{24}) = 24 = (3)(8) = (3^1)(8)$, and $3 \nmid 8$

$\Longrightarrow$ by first sylow theorem, the group $(Z_{24}, +_{24})$ has sylow 3- subgroup of order 3^1.

$\Longrightarrow$ $(\langle 8 \rangle, +_{24})$ is a sylow 3- Subgroup.

Theorem(3-7):

Let p a prime number and G be a finite group such that $p^x \backslash o(G), x \geq 1$, then G has a subgroup of order p^x which is called sylow p- subgroup of G.