

Advanced Agro-Hydro- Meteorology

A MSc course for students of Atmospheric Sciences

Dr. Thaer Obaid Roomi

2021-2022

Lecture 2: Precipitation (Part 1)

2.1 Introduction

Water may take a number of different forms in the atmosphere. These forms are collectively termed 'precipitation', which includes rain, drizzle, sleet (partly melted snowflakes, or rain and snow falling together), snow and hail. The intensity and duration of precipitation are extremely variable in most areas of the world. The source of precipitation is water vapor, which is always present in the atmosphere in varying amounts, although it makes up less than 1% by volume. However, the water vapor in the air must be cooled to allow water to be condensed into cloud droplets. These droplets then grow to form precipitation particles. The mass of water in the atmosphere in both liquid and vapor forms is around 1.3×10^{16} kg, compared with the mass of water in the oceans of around 1.3×10^{21} kg. Nevertheless this water is distributed very unevenly, and is transported by the circulation of the atmosphere. We will consider the basic thermodynamic processes which lead to the formation of precipitation, and describe the atmospheric systems within which these processes occur.

2.2 Equation of state for a perfect gas

A perfect gas (ideal gas) obeys the physical laws of Boyle and Charles. The gas equation (or equation of state) is:

$$p = \rho RT \quad (2.1)$$

where $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$ is known as the universal gas constant.

$$\text{Boyle's law:} \quad p_1 V_1 = p_2 V_2 \quad \text{at constant } T$$

$$\text{Charles' law:} \quad \frac{p_1}{T_1} = \frac{p_2}{T_2} \quad \text{at constant } V$$

where V is volume. Combining the two laws we get:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} = C$$

C is constant depends on the mass of gas ($=287 \text{ J kg}^{-1} \text{ K}^{-1}$ specific gas constant). These laws, are true for a perfect gas, and are nearly true for atmospheric gases.

Dalton's law states that in a mixture of gases, the total pressure is equal to the sum of the pressures which would be exerted by each gas if it filled the volume under consideration at the same temperature. Hence a mixture of gases, such as the atmosphere, behaves like a single gas provided the mixture gas constant R is given by:

$$R = \frac{\sum m_i R_i}{\sum m_i} \quad (2.2)$$

where m_i and R_i is the molecular weight and gas constant of the i th gas.

2.3 First law of thermodynamics

The first law of thermodynamics is derived from two facts: (i) heat is a form of energy; and (ii) energy is conserved. Conservation of energy Q may be expressed by

$$dQ = dU + dW \quad (2.3)$$

where U is the internal energy and W is the work done by the gas in moving from one state to another. Generally, terms of unit mass of gas:

$$dq = du + dw \quad (2.4)$$

$$dq = C_v dT + p d\alpha \quad (2.5)$$

$\alpha = \frac{1}{\rho}$ is specific volume. Differentiation of the equation of state ($p\alpha = RT$) gives:

$$p d\alpha + \alpha dp = R dT$$

$$p d\alpha = R dT - \alpha dp \quad (2.6)$$

Substitute (2.6) in (2.5) we get:

$$dq = C_v dT + R dT - \alpha dp$$

$$dq = (C_v + R) dT - \alpha dp$$

$$dq = C_p dT - \alpha dp \quad (2.7)$$

where $C_p = R + C_v = 1004 \text{ Jkg}^{-1}\text{K}^{-1}$ and $C_v = 717 \text{ Jkg}^{-1}\text{K}^{-1}$ are the specific heat at constants pressure and volume, respectively.

Both of equations (2.5) and (2.7) represent the first law of thermodynamics.

2.4 Atmospheric processes: dry adiabatic lapse rate

It is assumed that $dq=0$ for most air parcel movements. This assumption can be made whenever the motion is fast so that the heat exchange between the parcel and the surroundings is negligible. (Why?)

For adiabatic motion, equations of first law of thermodynamics become:

$$C_v dT + p d\alpha = 0 \quad (2.8)$$

$$C_p dT - \alpha dp = 0 \quad (2.9)$$

Solving for p in equation (2.9) from the equation of state ($p = \frac{1}{\alpha} R T$)

$$C_v dT + \frac{1}{\alpha} R T d\alpha = 0$$

$$C_v dT = -R T \frac{d\alpha}{\alpha}$$

$$C_v \frac{dT}{T} = -R \frac{d\alpha}{\alpha}$$

$$\int_{T_1}^T \frac{dT}{T} = -\frac{R}{C_v} \int_{\alpha_1}^{\alpha} \frac{d\alpha}{\alpha}$$

$$\ln (T - T_1) = -\frac{R}{C_v} (\ln(\alpha - \alpha_1))$$

Now, by taking the exponential (e) for the two sides:

$$\frac{T}{T_1} = \left(\frac{\alpha}{\alpha_1}\right)^{-\frac{R}{C_v}} \quad (2.10)$$

If T increases, α will decrease and vice versa.

$$\text{From equation of state:} \quad \alpha = \frac{R T}{p} \quad (2.11)$$

Substitute (2.11) in (2.10) and using (2.9) we get:

$$\begin{aligned} C_p dT - \frac{R T}{p} dp \\ \int_{T_1}^T \frac{dT}{T} = -\frac{R}{C_p} \int_{p_1}^p \frac{dp}{p} \\ \ln \frac{T}{T_1} = \frac{R}{C_p} \ln \frac{p}{p_1} \\ \frac{T}{T_1} = \left(\frac{p}{p_1}\right)^{\frac{R}{C_p}} \end{aligned} \quad (2.12)$$

If T increases P will increase and vice versa.

From equation (2.10) and equation (2.11) we get:

$$\begin{aligned} \left(\frac{\alpha}{\alpha_1}\right)^{-\frac{R}{C_v}} &= \left(\frac{p}{p_1}\right)^{\frac{R}{C_p}} \\ \frac{\alpha}{\alpha_1} &= \left(\frac{p}{p_1}\right)^{-\frac{C_v}{C_p}} \end{aligned}$$

An increase in p corresponds to a decrease in α and vice versa

From equation 2.12 and taking a reference pressure of 1000 millibars, we may define potential temperature θ as follows:

$$\theta = T \left(\frac{1000}{p}\right)^K \quad (2.13)$$

Where $K = R/C_p$. Therefore, θ is a constant for a dry adiabatic process, that is, one with no condensation or evaporation. Knowing that the hydrostatic equation is:

$$\frac{\partial p}{\partial z} = \frac{-g}{\alpha} \quad (2.14)$$

From equation (2.9), and differentiating with respect to height z , and using (2.14):

$$\begin{aligned} C_p \frac{dT}{dz} - \alpha \frac{dp}{dz} &= 0 \\ C_p \frac{dT}{dz} &= -g \\ \frac{dT}{dz} &= -g/C_p = \gamma_d = -9.76^\circ \text{C km}^{-1} \end{aligned} \quad (2.15)$$

Therefore if a parcel of air rises dry adiabatically, its temperature will fall at the rate of about $10^\circ \text{C km}^{-1}$. The quantity γ_d is the *dry adiabatic lapse rate* (DALR).

2.5 The Clausius-Clapeyron Equation

Clausius-Clapeyron equation calculates the change of the saturation vapor pressure with temperature (de_s/dT) during a phase change.

$$\frac{de_s}{dT} = \frac{L_{12}}{T(\alpha_2 - \alpha_1)} \quad (2.16)$$

If L_{12} , α_1 and α_2 are known functions of T , then (2.16) can be integrated to obtain a relation between saturation vapor pressure and T .

Since $\alpha_2 \gg \alpha_1$ (specific volume of water vapor is much greater than specific volume of liquid water or ice), equation 2.16 becomes:

$$\frac{de_s}{dT} = \frac{L_{12}}{T\alpha_2} \quad (2.17)$$

From the equation of state ($\alpha_2 = R_v T / e_s$) we get:

$$\frac{de_s}{e_s} = \frac{L_{12}}{R_v} \frac{dT}{T^2} \quad (2.18)$$

Therefore,

$$\ln\left(\frac{e_s}{e_{s0}}\right) = \frac{L_{12}}{R_v} \left(\frac{1}{T_0} - \frac{1}{T}\right) \quad (2.19)$$

Where e_{s0} is the saturation vapor pressure at T_0 .

For evaporation, $e_{s0} = 6.11$ mb and $T_0 = 273$ K, which implies mb and $T_0 = 273$ K, which implies

$$\ln\left(\frac{e_s}{6.11 \text{ mb}}\right) = \frac{L_{\text{evap}}}{R_v} \left(\frac{1}{273} - \frac{1}{T}\right) \quad (2.20)$$

$$\text{or } e_s = 6.11 \text{ mb} \exp\left[\frac{L_{\text{evap}}}{R_v} \left(\frac{1}{273} - \frac{1}{T}\right)\right] \quad (2.21)$$

For sublimation,

$$\ln\left(\frac{e_s}{6.11 \text{ mb}}\right) = \frac{L_{\text{sub}}}{R_v} \left(\frac{1}{273} - \frac{1}{T}\right) \quad (2.22)$$

$$\text{or } e_s = 6.11 \text{ mb} \exp\left[\frac{L_{\text{sub}}}{R_v} \left(\frac{1}{273} - \frac{1}{T}\right)\right] \quad (2.23)$$

Ex: Calculate the change in the melting point of ice if the pressure is increased from 1 to 2 atm, given that: $\alpha_i = 1.0908 \times 10^{-3} \text{ m}^3 \text{kg}^{-1}$, $\alpha_w = 1.001 \times 10^{-3} \text{ m}^3 \text{kg}^{-1}$

$$L_{\text{melt}} = 3.34 \times 10^5 \text{ Jkg}^{-1} \text{ at } 0^\circ \text{C}$$

Solution: Using Clausius-Clapeyron equation,

$$\begin{aligned} dT &= T(\alpha_w - \alpha_i) \frac{dp}{L_{\text{melt}}} \\ &= 273 \times (1.001 - 1.0908) \times 1.013 \times 10^5 / 3.34 \times 10^5 = -0.00744 \text{ deg} \end{aligned}$$

Therefore, an increase in pressure of 1 atm decreases the melting point of ice by about 0.007 deg. Usually, the melting point increases with increasing pressure. But ice is unusual because $\alpha_w < \alpha_i$.

Equations (2.20) and (2.22) plus a similar equation for melting allow us to plot the curves of saturation pressure versus temperature. These curves describe the T - e relations during phase changes ($e=e_s$).

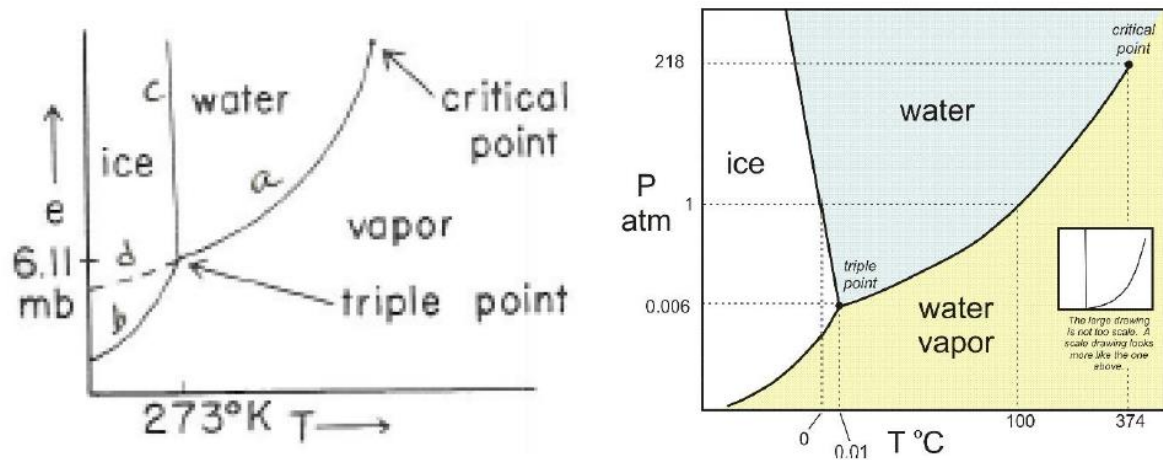


Figure 2.1 T , e phase diagram for water substance.

All curves begin at the triple point.

(a) **Evaporation curve a**: curves upward exponentially to the right according to Eq. (2.20) ($T > 273$ K, and $e_s > 6.11$ mb). Along this curve, water and vapor are in equilibrium until it reaches the critical point ($T = 374$ K) where only vapor can exist. At some point along this curve, water starts to boil. That is the point when $e_s = p_{atm}$.

(b) **Sublimation curve b**: curves downward exponentially to the left of the triple point ($T < 273$ K, $e_s < 6.11$ mb). This curve is steeper than the evaporation curve because $L_{sub} > L_{evap}$.

(c) **Melting curve c**: since $\alpha_w - \alpha_i$ is almost zero (a very small negative number), de_s/dT is almost $-\infty$, based on the Clausius-Clapyeron equation. Thus the curve is almost vertical, but tilts very slightly to the left.

(d) **Supercooled water d**: When water which does not freeze below 273 K (0°C), it is called supercooled water. For supercooled water, within a certain temperature range of $T < 0^\circ\text{C}$, $e_{sw} > e_{si}$ (curve d).

Therefore, in a cloud with mixture of ice crystals and supercooled water (cloud) droplets, water vapor will first condense on the ice because e_{si} is smaller than e_{sw} .

In other words, in the competition for water vapor, ice will win over supercooled water. This leads to the well-known **Bergeron-Findeison process**. That is, ice crystal in a cloud of supercooled water droplets will grow faster than the water droplets.

2.6 Atmospheric processes: saturated adiabatic lapse rate

For a sample of moist air in which no evaporation or condensation occurs, equation 2.16 may be used. However, when condensation occurs and the resulting water falls out of the sample, the mass of the sample changes and heat is lost with the fallout of the water. This is known as a pseudo-adiabatic process. If all the condensed water remains in the

sample, then the process is of course reversible. In the atmosphere conditions are usually such that some, but not all, of the condensed water falls out of any sample of moist air. Using Eqs 2.1, 2.9 and 2.16, it may be shown that for saturated air which is lifted slightly,

$$\frac{\partial T}{\partial Z} = \gamma_s = \frac{\gamma_d \left(1 + \frac{L x_s}{R_d T}\right)}{1 + \frac{L^2 x_s}{R' C_p T^2}} \quad (2.17)$$

where γ_s is the *saturated adiabatic lapse rate* (SALR) and x_s is the *saturated humidity mixing ratio*, that is, the mass of water vapor present in the moist air measured per gram of dry air when the moist air is saturated. Although γ_s varies with temperature and pressure, a typical value in the atmosphere is $-5.0 \text{ }^\circ\text{C km}^{-1}$.

2.7 Stability and convection in the atmosphere

Moist air can become saturated, and hence produce precipitation, by movement upwards in the atmosphere. Consider a parcel of moist air, for which the pressure in the parcel is the same as that of its environment. Assuming that the parcel can move vertically without disturbing the environment and does not mix with its environment, it can be shown from the equations of motion for the atmosphere that

$$\frac{dw}{dt} = -\frac{\partial p}{\partial Z} - g = \frac{(T - T')g}{T'} \quad (2.18)$$

where w is the vertical velocity of the parcel, t is time, T is the temperature of the parcel, and T' is the temperature of the environment. Making the further assumption that the movement is adiabatic,

$$\frac{dw}{dt} = \frac{(T_0 - T'_0)g}{T'_0} + \frac{(\gamma - \gamma_a)g z}{T'_0} \quad (2.19)$$

where T_0 and T'_0 are the initial temperature of the parcel and the environment respectively, γ is the environmental lapse rate, z is the vertical coordinate, and γ_a is the appropriate adiabatic lapse rate, being γ_d if the parcel is unsaturated and γ_s if the parcel is saturated. If the temperature is constant in the horizontal then $T_0 = T'_0$ and

$$\frac{dw}{dt} = \frac{(\gamma - \gamma_a)g z}{T'_0} \quad (2.20)$$

The atmosphere is regarded as stable, neutral or unstable when dw/dt is <0 , 0 or >0 , respectively. If $\gamma_s < \gamma < \gamma_a$ then the atmosphere is *conditionally unstable*, whereas if $\gamma < \gamma_s$ the atmosphere is *absolutely stable* and if $\gamma > \gamma_d$ the atmosphere is *absolutely unstable*. Hence if moist air is lifted by some means it may become saturated and hence unstable, and may then continue to rise without any external force being applied.

2.8 The growth of precipitation particles and Bergeron Process

- The condensation of water vapor in the atmosphere, brought about by the movement of air upwards, provides water droplets or ice crystals in clouds. Such precipitation particles are denser than the air surrounding them, and therefore they begin to fall at a rate of a few centimeters per second. However, these particles will either evaporate in unsaturated air below the cloud, or be held suspended by

vertical currents within the cloud. They will only be able to reach the ground as precipitation if they become large enough to stand evaporative losses and overcome upward air motions.

- In order for cloud droplets, which are very small, to become rain drops, they have to increase in size almost a million times. Indeed, for even a cloud droplet to form, complicated processes must take place allowing for the conversion of water vapor to liquid water.
- Often times in the atmosphere this process would be virtually impossible without the presence of aerosols. Before we look at this process involving CCN, or cloud condensation nuclei, let us first examine the case without them, known as homogeneous nucleation.
- We have said before that the process of the change of state from vapor to liquid is called condensation. Also, this will occur when the relative humidity reaches 100%, or when the vapor pressure equals the saturation vapor pressure.
- In the microphysics of clouds condensation, however, pure water will condense only when levels of saturation reach upwards of 120% (20% supersaturation). The reason being that the spherical shape a water droplet forms is a very unstable structure, hence resisting formation of the droplet. It is not until these high levels of saturation are reached that the forcing will overcome this resistance known as surface tension.
- The process known as heterogeneous nucleation involves "polluting" the pure water with aerosols, or CCN. By adding CCN, water is allowed to condense with much lower values of supersaturation, on the order of a few tenths of a percent.
- Now that cloud droplets have formed, we will try to understand how they can grow to the size of a raindrop.
- One such way (although, as we will soon see, not the most important) is through collision and coalescence. Cloud droplets will be carried by air currents within the cloud, and if they bump into each other, it is called a collision.
- However, if they collide then stick together, that is called coalescence. Although this process is important, especially in the tropics and in increasing the size of raindrops, it falls short of being the primary mechanism for the formation of raindrops. The process needed was serendipitously discovered by a man named Tor Bergeron while taking a mountain walk.
- The Bergeron process relies primarily on the fact that the saturation vapor pressure with respect to ice is less than the saturation vapor pressure with respect to water.
- Another important fact is that pure water droplets do not freeze at 0°C! Again, because of surface tension and the structure of water, to get a pure water droplet to freeze requires a temperature of -40°C.
- Liquid water that is cooler than 0°C is called supercooled. In the atmosphere, similar to CCN, there exist freezing nuclei. In contrast to CCN, freezing nuclei are not plentiful in the atmosphere because their structure must be similar to the structure of an ice crystal. Most of the naturally occurring freezing nuclei

"activate" at about -10°C . These freezing nuclei allow for the cloud droplets to freeze around them.

- Because of the relative sparseness of the freezing nuclei, ice crystals and supercooled water droplets can coexist at the same time. This is where the Bergeron's primary fact becomes important.
- The following chart illustrates the differences in saturation vapor pressures of water.

Temperature	RH wrt* $\text{H}_2\text{O}(\text{liq})$	RH wrt $\text{H}_2\text{O}(\text{ice})$
0°C	100%	100%
-05°C	100%	105%
-10°C	100%	110%
-15°C	100%	115%
-20°C	100%	121%
*wrt = with respect to		

Note that since $\text{RH} = e/e_s$, if e_s is made smaller, RH increases.

- The Bergeron process can be summarized as such: The air reaches saturation and some of the resulting droplets will come in contact with freezing nuclei (assuming they have reached the activation temperature).
- We will now have a combination of ice crystals and supercooled water droplets. From the perspective of the supercooled droplets, the air is in equilibrium at saturation, but from the perspective of the ice crystals, the air is supersaturated.
- Therefore, water vapor will sublime on the ice crystals. Since the amount of water vapor in the air has decreased, and from the perspective of the supercooled water droplet, the air is subsaturated, the supercooled water will evaporate until the air once again reaches saturation. The process then continues.
- In short summary, the ice crystal grows through sublimation at the expense of the supercooled water droplet.